

# Basic Probability



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## Introduction

This book provides a brief introduction to some common ideas in the study of probability. At the University of Minnesota, this material is included in a course on College Algebra designed to give students the basic skills to take an introductory Statistics course. The material itself is basic, and should be within the grasp of students who have successfully completed a high school Algebra I course. It comprises approximately three weeks worth of material at the college level; a typical college student would spend about 45 hours total learning this material.

These materials are used as part of an active learning course, and the interplay between in-class activities that allow students to explore and discover ideas and a more formulaic study of the necessary computations is important. These lessons address primarily the second objective, the skill-building component of mathematics. Can you successfully carry out the calculations necessary to find answers to a variety of applications? Conceptual understanding of the material is also important, and instructors are encouraged make use of activities that help students develop higher-order thinking skills which advance the computational skills developed here.

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Mike Weimerskirch

Director of Educational Innovation

Univ. of Minnesota Math Center for Educational Programs





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# Chapter 1 – Counting Techniques

## Introduction

This chapter introduces students to counting techniques necessary for the study of probability. How many outcomes are possible when rolling two dice? when flipping four coins? when drawing five cards from a deck of 52? The basic building block is the General Counting Principle, from which Permutations are a specific application. The connection between Permutations and Combinations is developed, as well as the connection between Binary models (coin flips) and Combinations. Seeing the interconnectedness of these topics is important for success in probability and students should be encouraged to focus on the similarities between topics. For example, why is the formula for combinations the same as the formula for permutations with an extra factor in the denominator? What is the extra factor? What does it represent?

The lesson on Permutations is presented in two parts. Part I follows the General Counting Principle and places ALL members of a set in order. Part II does the generalization when only some of the members are placed in order.

## Chapter Topics

- Topic 1.1 – General Counting Principle
- Topic 1.2 – Binary Counting
- Topic 1.3 – Permutations
- Topic 1.4 – Permutations II
- Topic 1.5 – Combinations



## Topic 1.1 – General Counting Principle

This lesson is the first of five lessons on the counting techniques needed for a study of probability. The *General Counting Principle*, also known as the **Multiplication Principle**, is the foundation for the lessons in *Binary Counting* and *Permutations – Parts I and II*. It can be done fairly quickly, as students generally don't appreciate the technique's power until dealing with *Binomial Probabilities* and *Permutations*. It is however, an important foundational technique that should be referred to in these future topics.

The key is to get students to realize that in a two-step process, the list of possible outcomes forms a two-dimensional grid.

Do something for the video

Standard Deck of Cards

K♠	K♥	K♦	K♣
Q♠	Q♥	Q♦	Q♣
J♠	J♥	J♦	J♣
10♠	10♥	10♦	10♣
9♠	9♥	9♦	9♣
8♠	8♥	8♦	8♣
7♠	7♥	7♦	7♣
6♠	6♥	6♦	6♣
5♠	5♥	5♦	5♣
4♠	4♥	4♦	4♣
3♠	3♥	3♦	3♣
2♠	2♥	2♦	2♣
A♠	A♥	A♦	A♣

University of Minnesota General Counting Principle

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Transcript

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## Topic 1.2 - Binary Counting

This lesson extends the *General Counting Principle* to the specific case where each choice is an either/or choice. The basic model is that of flipping a sequence of coins. It is a precursor to lessons in *Combinations* and in *Binomial Probabilities*.



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## Topic 1.3 - Permutations

This lesson is the first of two parts dealing with *Permutations*. This first part deals with placing all members of a set in order. The generalization to ordering only some of the members of a set is saved for *Part II*. The topic introduces factorials as the basic computational technique for permutations.



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## Topic 1.4 - Permutations II

This lesson expands the usefulness of the permutation counting technique developed in the *Permutations – Part I* lesson to include permutations of a partial list of the full set of elements from which we are choosing. (For example, list the 1st, 2nd, and 3rd place winners in a eight horse race.) It gives the standard formula for permutations:

$$P(n, k) = \frac{n!}{(n-k)!}$$

It is important to note that the process for counting permutations relies on the *General Counting Principle*, which involves only multiplication. The division in the formula is not natural. It is important conceptually for students to understand the underlying multiplication, and not just rely on the formula.



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## Topic 1.5 - Combinations

This lesson develops an understanding of *Combinations* as they relate to *Permutations*. The connection between ordered lists (*Permutations*) and unordered sets (*Combinations*) is of fundamental importance and time should be spent developing this connection. The recursion of Pascal's Triangle and its connection to the recursion for *Combinations* is presented here. That connection is also of fundamental importance in understanding probability. This idea is also used in the standard formula for calculating binomial probabilities (*i.e.* calculating using the Binomial Distribution).



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## Chapter 2 - Probability

This chapter introduces students to the basics of probability. The emphasis is on problems that occur naturally, both in the playing of games and in natural phenomena. The Binomial model is stressed, as many problems arise from a sequence of either/or choices. The Bell Curve is of fundamental importance in Statistics and is addressed here.



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## Topic 2.1 - Basic Definitions of Probability

*Basic Definitions of Probability* is the first in a series on lessons developing the foundations of probability theory. It defines **events**, establishes probability for equally likely outcomes (the ‘equiprobable model’) and gives a brief example. Further examples appear in later lessons.



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## Topic 2.2 - And, Or, Not

This lesson introduces the methods for calculating probabilities of events combined using intersections, unions and complements. The formality of set theory is not discussed. Instead, we rely on a more informal approach, though the formal symbolism appears alongside the more intuitive description. Only a few brief examples are given to illustrate the concepts. Students need to be exposed to a great variety of problems to fully appreciate the power of these techniques.



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## Topic 2.3 - Random Variables

This brief lesson introduces the concept of a **random variable**, and gives a couple of brief examples. Further examples appear in later lessons.



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## Topic 2.4 - Expected Value

This lesson introduces the concept of **expected value**. It may be helpful to be familiar with random variables, but this lesson can be done with that formal description.



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## Topic 2.5 - Binomial Probabilities I

This lesson, **Binomial Probabilities – Part I** is the first of two lessons calculating probabilities of repeated trials with outcomes that have only two possibilities, such as flipping a coin. This lesson deals with the ‘fair’ case where the probabilities are each 50%. A future lesson, Binomial Probabilities – Part II expands this to arbitrary probabilities. The connection to Pascal’s Triangle is important, and it is assumed that students have familiarity with Pascal’s Triangle. The recursion for Pascal’s Triangle, and its connection to counting techniques is embedded in the lesson and is of fundamental importance to understanding this concept.



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## Topic 2.6 - Binomial Probabilities II

This lesson is the second of two lessons calculating probabilities of repeated trials with outcomes that have only two possibilities. This lesson expands the ‘fair coin’ model to arbitrary probabilities. Advanced courses in Probability will call this the ‘Binomial Distribution’, and the standard formula is given, as well as several examples.



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## Topic 2.7 - Bell Curve

This lesson discussed the *bell curve* as a limit, but without the formalities necessary to truly talk about the *Central Limit Theorem*. The lesson begins with some *binomial distributions* to develop an understanding of where the bell curve comes from. The standardization process (z-scores) are discussed with examples. Additionally, examples involving confidence intervals are presented, though no attempt is made to discuss the formalities of *Hypothesis Testing*.



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## Topic 2.8 - Bayes' Rule

This lesson calculates condition probabilities in a specific example, that of assessing risk of disease in the setting where there is a diagnostic test that has some level of error. The intuition behind **Bayes' Rule** is presented, but no formal notation is used.

### Bayes Rule

pop. = 10000	Tested positive	Tested negative	
infected	99%	1%	2%
not infected	3%	97%	98%
Total			

University of Minnesota Bayes Rule and Testing

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