

Linear Speed and Angular Speed

Preliminaries and Objectives

Preliminaries:

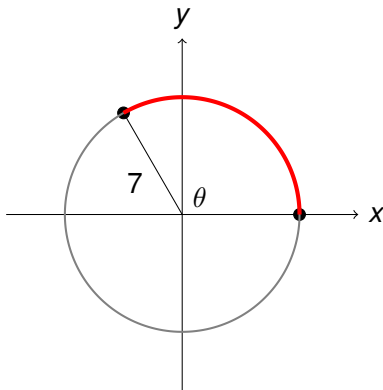
- Circumference of a circle
- Conversion factors (dimensional analysis)

Objectives:

- Given the central angle and radius (or diameter) of a circle, find the arc length
- Given the rotational speed of a tire, find the linear speed of the vehicle.

Finding arc length from radian measure

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Find the length of the arc of a circle of radius = 7, where the central angle measures 120°

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Arc length = (fraction of circle) x (circumference of circle)

$$C = 2\pi r = 14\pi$$

$$\text{Arc length} = \frac{120^\circ}{360^\circ}(14\pi) \approx 14.661$$

Finding distance travelled from angle measure

Distance moved in one revolution = circumference of circle

$$\text{Distance moved in one revolution} = 2\pi r = \pi d$$

$$\text{Angle moved in one revolution} = 360^\circ = 2\pi \text{ radians.}$$

Finding linear speed from angular speed

A car tire has a diameter of 25 inches. If the tire is rotating at 10 revolutions per second, how fast is the car traveling?

Solution:

$$d = 25 \Leftrightarrow C = \pi d = 25\pi \text{ inches} \approx 78.54 \text{ inches}$$

$$10 \frac{\text{rev.}}{\text{sec.}} \approx 10(78.54 \frac{\text{in.}}{\text{sec.}}) \approx 785.4 \frac{\text{in.}}{\text{sec.}}$$

$$\frac{785.4 \text{ in.}}{\text{sec}} \cdot \frac{1 \text{ ft.}}{12 \text{ in.}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft.}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 44.6 \text{ mi./hr.}$$

Finding distance travelled from angle measure

A bike tire has a diameter of 35 inches. If the tire rotates 10 revolutions, how far did the bicycle travel?

Distance moved in one revolution = $\pi d = 35\pi \approx 109.956$ inches

$$10 \text{ revolutions} = 10(35\pi) \approx 1099.56 \text{ inches}$$

$$1099.56 \text{ inches} \cdot \frac{1 \text{ ft.}}{12 \text{ in.}} \approx 91.63 \text{ ft.}$$

Finding angular speed from linear speed

A tractor is traveling 8 mph. If the tractor tire has a radius of 40 inches, how fast is the tire rotating?

Solution:

$$r = 40 \Leftrightarrow C = 2\pi r = 80\pi \text{ inches} \approx 251.33 \text{ inches}$$

$$\frac{8 \text{ mi.}}{\text{hr.}} \cdot \frac{5280 \text{ ft.}}{\text{mi.}} \cdot \frac{12 \text{ in.}}{\text{ft.}} \cdot \frac{\text{hr.}}{60 \text{ min.}} \cdot \frac{1 \text{ rev.}}{251.33 \text{ in.}} \approx 33.6 \text{ rev./min.}$$