Linear Speed and Angular Speed

Preliminaries and Objectives

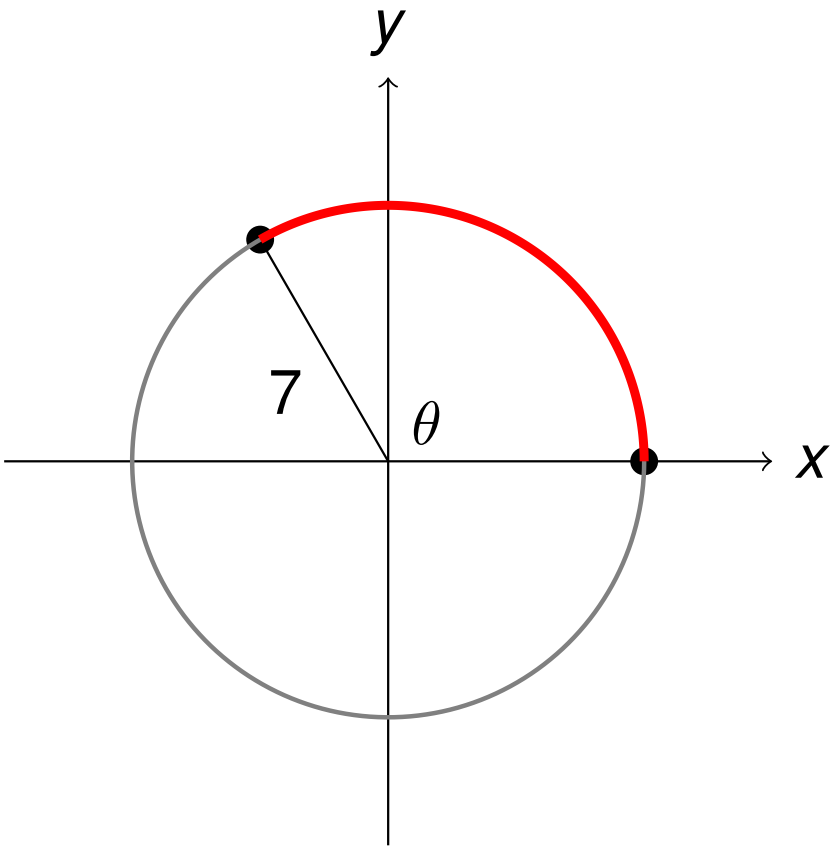
Preliminaries:

- Circumference of a circle
- Conversion factors (dimensional analysis)

Objectives:

- Given the central angle and radius (or diameter) of a circle, find the arc length
- Given the rotational speed of a tire, find the linear speed of the vehicle.

Finding arc length from radian measure



Find the length of the arc of a circle of radius $= 7$, where the central angle measures 120°

Finding arc length from radian measure

Find the length of the arc of a circle of radius $= 7$, where the central angle measures 120°

Arc length = (fraction of circle) x (circumference of circle)

$$C = 2\pi r = 14\pi$$

$$\text{Arc length} = \frac{120^\circ}{360^\circ}(14\pi) \approx 14.661$$

Finding distance travelled from angle measure

Distance moved in one revolution = circumference of circle

Distance moved in one revolution $= 2\pi r = \pi d$

Angle moved in one revolution $= 360^\circ = 2\pi$ radians.

Finding distance travelled from angle measure

A bike tire has a diameter of 35 inches. If the tire rotates 10 revolutions, how far did the bicycle travel?

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$$1099.56 \text{ inches} \cdot \frac{1\text{ft.}}{12\text{in.}} \approx 91.63 \text{ ft.}$$

Finding linear speed from angular speed

A car tire has a diameter of 25 inches. If the tire is rotating at 10 revolutions per second, how fast is the car traveling?

Finding linear speed from angular speed

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Solution:

$d = 25 \Leftrightarrow C = \pi d = 25\pi$ inches ≈ 78.54 inches

$10\frac{\text{rev.}}{\text{sec.}} \approx 10(78.54\frac{\text{in.}}{\text{sec.}}) \approx 785.4\frac{\text{in.}}{\text{sec.}}$

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$$\frac{785.4\ in.}{sec} \cdot \frac{1\ ft.}{12\ in.} \cdot \frac{1\ mile}{5280\ ft.} \cdot \frac{60\ sec}{1\ min} \cdot \frac{60\ min}{1\ hr} \approx 44.6\ mi./hr.$$

Finding angular speed from linear speed

A tractor is traveling 8 mph. If the tractor tire has a radius of 40 inches, how fast is the tire rotating?

Finding angular speed from linear speed

A tractor is traveling 8 mph. If the tractor tire has a radius of 40 inches, how fast is the tire rotating?

Solution:

$r = 40 \Leftrightarrow C = 2\pi r = 80\pi$ inches ≈ 251.33 inches

$$\frac{8\ mi.}{hr.} \cdot \frac{5280\ ft.}{mi.} \cdot \frac{12\ in.}{ft.} \cdot \frac{hr.}{60\ min.} \cdot \frac{1\ rev.}{251.33\ in.} \approx 33.6\ rev./min.$$

Credits

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