Sequences and Recursion

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Preliminaries and Objectives

Preliminaries

- Functions and function notation

Objectives

- Define sequences from an explicit formula
- Define sequences from a recursive formula

Sequences

A **sequence** is an ordered set of numbers. Typically a sequence is denoted

$$\{a_n\} = \{a_1, a_2, a_3, \ldots\}$$

where the subscript indicates the term in the sequence.

Sequences

A **sequence** is an ordered set of numbers. Typically a sequence is an infinite list.

Example 1

The positive even numbers $= \{2, 4, 6, 8, 10 \ldots\}$

Example 1

$$E = \{2, 4, 6, 8, 10 \ldots\}$$

$$\{e_n\} = \{2, 4, 6, 8, 10 \ldots\}$$

$$e_1 = 2, e_2 = 4, e_3 = 6, e_4 = 8$$

$$e_{24} = 48$$

$$e_n = 2n$$

Sequences and Functions

$$\{e_n\} = \{2, 4, 6, 8 \ldots\}$$

Sequence Notation: $e_n = 2n$

Function Notation: $f(x) = 2x$

Equation of a Line: $y = 2x$

Example 2

$$m_n = 2^n - 1$$

$$\{m_n\} = \{1, 3, 7, 15, 31 \ldots\}$$

- Define the first term
- Give a formula to determine the next term from the previous term

The positive even numbers $= \{e_n\} = \{2, 4, 6, 8 \ldots\}$ can be determined from the recursive formula

$$\begin{aligned} e_1 &= 2 \\ e_{n+1} &= e_n + 2 \end{aligned}$$

Example 3 - Compound Interest

How much money will you have at the end of each year if you deposit \$100 at 6% interest, compounded annually?

$$\{d_n\} = \{\$106.00, \$112.36, \$119.10 \ldots\}$$

$$d_0 = 100$$
$$d_{next} = 1.06 \cdot d_{prev}$$

$$d_n = 100(1.06)^n$$

How much money will you have at the end of 50 years?

$$d_{50} = \$100(1.06)^{50} = \$1842.02$$

Example 4

$$a_1 = 1$$
$$a_{n+1} = 3a_n - 1$$

$$\{a_n\} = \{1, 2, 5, 14, 41, 122 \ldots\}$$

$$a_n = ???$$

Example 5 - Fibonacci Sequence

$$f_1 = 1, f_2 = 1, f_{n+2} = f_{n+1} + f_n$$

$$\{f_n\} = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots\}$$

Example 6

$$c_n = (-1)^n \cdot n$$

$$\{c_n\} = \{-1, 2, -3, 4, -5, 6 \ldots\}$$

Recap

- Explicit definition is formula, much like a function.
- Recursion defines the first term (or first few terms) and a method to calculate later terms based on previous terms.