1. Sigma Notation

2. You should be familiar with sequences of numbers.

   In this lesson, you will learn how to find a sum written using Sigma Notation.

3. (a) Sometimes we wish to find the sum of several terms of a sequence. For example, we may wish to find the sum of the even numbers up to 20. This is a long list, and it would be convenient to have a short hand way of writing it.

   (b) If the pattern is obvious, we often use dot-dot-dot to omit the middle terms, but now it is not clear how many terms are in the series. There is a more precise notation we can use, called Sigma Notation.

   (c) Sigma is the greek letter for s, which is the first letter in the word sum. The sigma indicates that we should add something up. The k in this case is called the index. It is like a variable in an explicit formula, the k indicates that we will be plugging in various values of k into a formula. 1 is called the lower summand. It tells us the first number to be plugged into the formula. Above the sigma is the upper summand, it tells us the last number to plug into the formula. It is common to use 1 as the lower summand. We always go up by one, until reaching the upper summand. In this case, we will plug in the numbers 1,2,3, up to 10. Since we go up by one, we always use integers in the summands, never fractions or decimals. It is sometimes useful to have the lower summand equal 0, and occasionally some other value but 1 and 0 are most common. Finally, we have the formula

   In this case, we first plug one into the formula and get two, then plug in two

   (d) and so on. Notice we did not stop when we got the number 10 from the formula. We plugged 5 into the formula and got ten. We continue until we have plugged 10 as the value for k in to the formula, getting 20, for a total of 10 terms. The Sigma then directs us to add the ten terms together, yielding 110 as the answer.

   (e) We could get the same series by writing the sigma notation in a different way. When we plug zero into this formula, we get 2. When we plug in $k = 1$, we get $2(1 + 1)$, which is 4. In fact, we get precisely the same sequence of numbers to add.

4. (a) Here is another example. We are asked to add five numbers together. The first number is 1^2, the second is 2^2, which is 4, up to the last number which is 25.

   (b) The sum is 55.

5. (a) Here is an example for you to try. Start by plugging in $k = 0$ into the formula, then $k = 1$ and so on until $k = 3$. Then add the terms together. You may wish to pause the video at this point to work out his problem.

   (b) The sum of the four numbers is 22.

6. (a) Sometimes, the upper summand is also a variable. We plug in the values from one up to some number n, which we can change.

   (b) Plugging in one, we get one, plug in two, we get three. Continue until plugging in the final value of n. When we plug n into the formula, we get $2n - 1$

   (c) For example, if n is four, then we add up the numbers 1,3,5 and 7, to get 16.

   (d) You may have noticed that the answer is always n^2. In general, it may be hard to find a pattern, and even harder to prove that the pattern is correct.

   (e) (*Graphic inserted here.*)

7. To recap: The Sigma directs you to find a sum, that is, to add up a bunch of numbers. The index indicates the various values that are plugged into the formula. The index is always an integer that goes up by one.