Solving Systems of Non-linear Equations

Preliminaries and Objectives

Preliminaries

- Graph of circles, ellipses, parabolas and hyperbolas.
- Transformation of Graphs
- Solving polynomial equations in one variable.

Objectives

- Find the intersection points of polynomial equations.

Solving Polynomial Equations by Factoring

$$4x^2 + x - 14 = 0$$

$$(4x - 7)(x + 2) = 0$$

$$4x - 7 = 0 \text{ or } x + 2 = 0$$

$$x = \frac{7}{4} \text{ or } x = -2$$

Solving Polynomial Equations by Factoring

$$x^4 - 25x^2 + 144 = 0$$

$$(x^2 - 9)(x^2 - 16) = 0$$

$$(x - 3)(x + 3)(x - 4)(x + 4) = 0$$

$$x = \pm 3 \text{ or } x = \pm 4$$

Solving Polynomial Equations by Completing the Square

$$(x-3)^2 = 7$$

$$x - 3 = \pm\sqrt{7}$$

$$x = 3 \pm \sqrt{7}$$

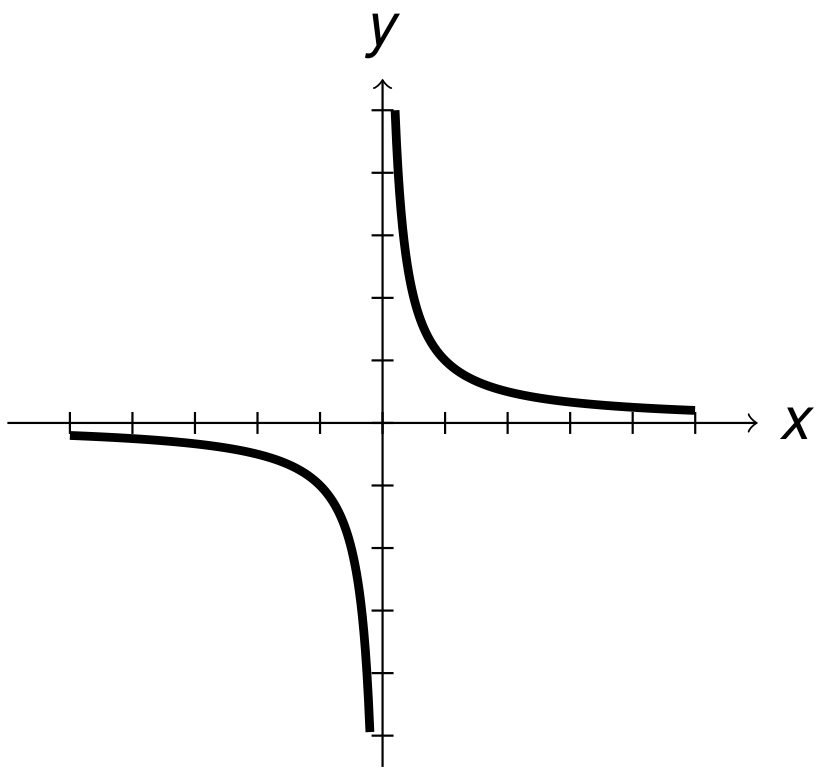
Solving Polynomial Equations by the Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

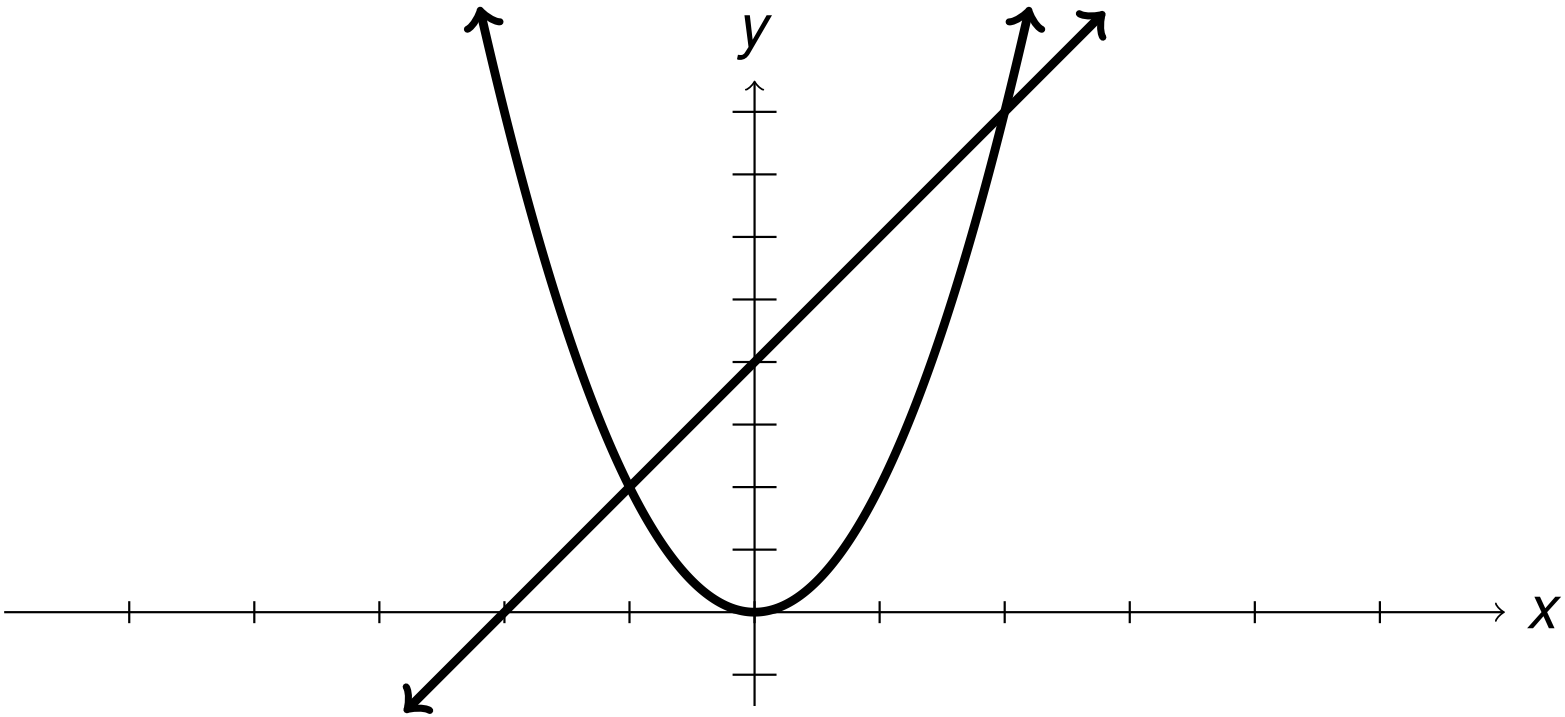
Hyperbola - Version 2

$$xy = 1 \Leftrightarrow y = \frac{1}{x}$$



Example 1 - Substitution

$$y = 2x^2$$
$$y = 2x + 4$$



Example 1 - Substitution

$$y = 2x^2$$
$$y = 2x + 4$$

$$2x^2 = 2x + 4$$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

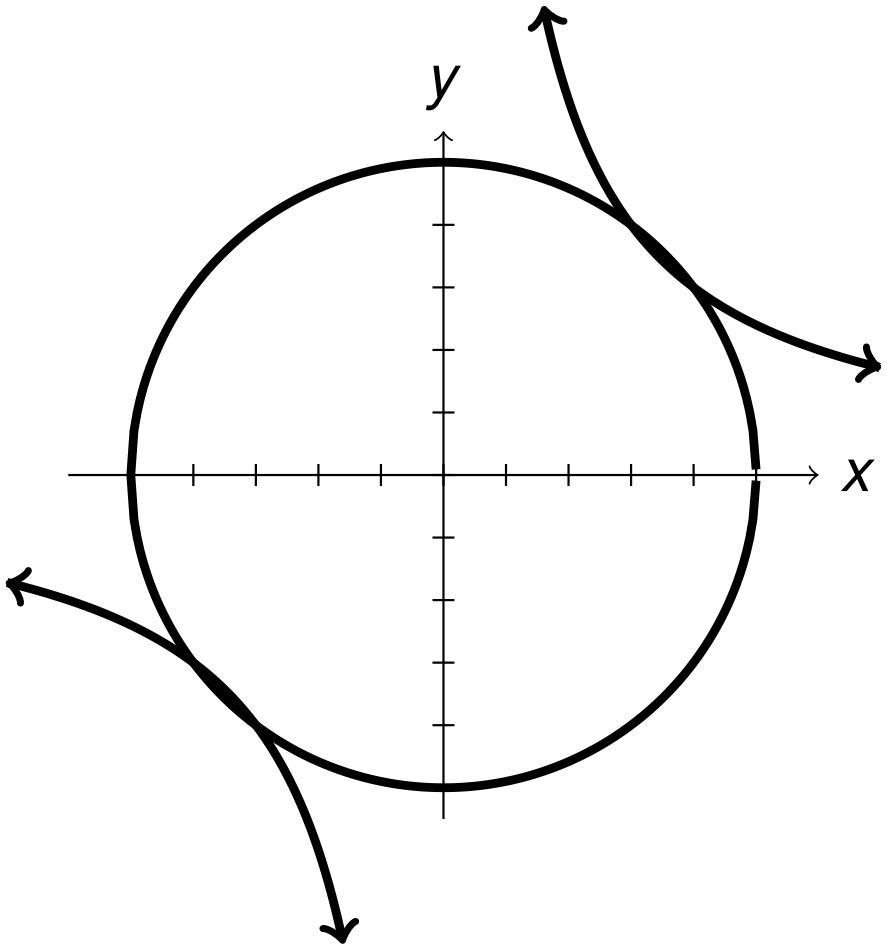
$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

$$(2, 8) \qquad (-1, 2)$$

Example 2 - Substitution

$$xy = 12$$
$$x^2 + y^2 = 25$$



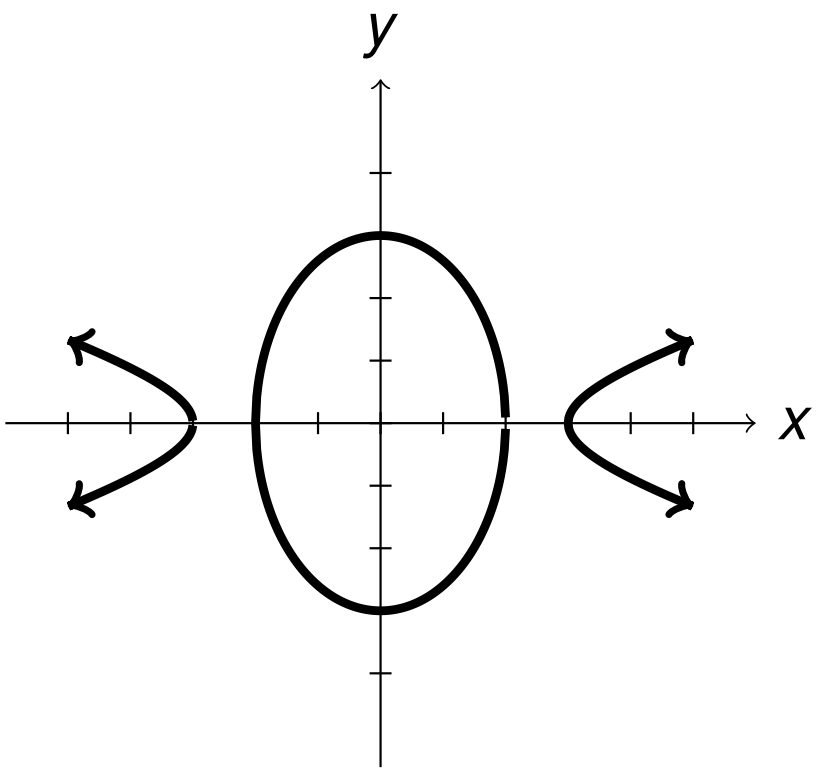
Example 2 - Substitution

$$xy = 12$$
$$x^2 + y^2 = 25$$

$$y = \frac{12}{x}$$
$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$
$$x^2 + \frac{144}{x^2} = 25$$
$$x^4 + 144 = 25x^2$$
$$x^4 - 25x^2 + 144 = 0$$
$$(x-3)(x+3)(x-4)(x+4) = 0$$
$$(3,4), (4,3), (-3,-4), (-4,-3)$$

Example 3 - Elimination

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$
$$\frac{x^2}{9} - y^2 = 1$$



Example 3 - Elimination

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$
$$\frac{x^2}{9} - y^2 = 1$$

$$\frac{9x^2}{4} + y^2 = 9$$

$$\frac{85x^2}{36} = 10$$

$$x = \pm\sqrt{\frac{72}{17}}$$

$$y^2 = -\frac{9}{17}$$

Example 4 - Substitution

$$4x^2 + y^2 = 16$$
$$y^2 = x + 2$$

$$4x^2 + x + 2 = 16$$

$$4x^2 + x - 14 = 0$$

$$(4x - 7)(x + 2) = 0$$

$$x = \frac{7}{4} \text{ or } x = -2$$

$$(-2, 0), \left(\frac{7}{4}, \frac{\sqrt{15}}{2}\right), \left(\frac{7}{4}, -\frac{\sqrt{15}}{2}\right)$$

Example 5 - Substitution

$$y = \sqrt{x}$$
$$y = x - 2$$

$$\sqrt{x} = x - 2$$

$$x = x^2 - 4x + 4$$

$$x^2 - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0$$

$$x = 4 \text{ or } x = 1$$

$$(4, 2), (1, -1)$$

Recap

- Substitution and Elimination techniques may be used
- Reduce the equation to a single variable
- Find all solutions for the first variable
- Substitute to find all ordered pairs
- Check solutions by graphing