Solving 3 x 3 Systems of Linear Equations

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Preliminaries and Objectives

Preliminaries

- Solving a 2 x 2 system of linear equations
  - Substitution Method
  - Elimination Method

Objectives

- Find the solution to a system of 3 equations in three variables.

Solving 2 x 2 Systems of Equations - Elimination Method

- Multiply one or both equations by a constant so that one variable will cancel.
- Add equations together to get new equation with one variable.
- Solve for first variable.
- Substitute to find second variable.

Solving 3 x 3 Systems of Equations

- Pick two of the three equations and multiply one or both equations by a constant so that one variable will cancel.
- Add equations together to get new equation with two variables.
- Pick a different pair of equations and multiply one or both equations by a constant so that the **same** variable will cancel.
- Add equations together to get new equation with the **same** two variables.
- Solve the new 2 x 2 system

Example 1

$$\begin{array}{lrcrcrcr}
(I) & x & + & y & + & z & = & 0 \\
(II) & -2x & + & 2y & - & 4z & = & 12 \\
(III) & 2x & - & 3y & - & z & = & 7
\end{array}$$

$$\begin{array}{lrcrcrcr}
2*(I) & 2x & + & 2y & + & 2z & = & 0 \\
(II) & -2x & + & 2y & - & 4z & = & 12 \\
\hline
(IV) & & & 4y & - & 2z & = & 12
\end{array}$$

$$\begin{array}{lrcrcrcr}
(V) & & - & y & - & 5z & = & 19
\end{array}$$

Example 1

$$\begin{array}{lrcrcl}
(IV) & & 4y & - & 2z & = & 12 \\
(V) & - & y & - & 5z & = & 19
\end{array}$$

$$\begin{array}{lrcrcl}
(IV) & & 4y & - & 2z & = & 12 \\
4*(V) & - & 4y & - & 20z & = & 76 \\
\hline
 & & & - & 22z & = & 88
\end{array}$$

$$\begin{array}{lcccccccl}
 & & & & & & z & = & -4 \\
(V) & & & - & y & + & 20 & = & 19 \\
 & & & & y & & & = & 1 \\
(I) & x & & + & y & + & z & = & 0 \\
(I) & x & & + & 1 & - & 4 & = & 0 \\
 & x & & & & & & = & 3
\end{array}$$

$(3, 1, -4)$

Geometry of Linear Equations in 3-dimensional space

- Each linear equation is the equation of a plane.
- Two planes intersect in a line (usually).
- A line intersects the third equation (plane) at a point (usually).
- A false statement means two planes were parallel and there are no points of intersection of all three planes.
- A true statement means that there are infinitely many solutions, either because two planes were the same plane or that every pair of planes intersects at the same line.

Example 2

$$\begin{array}{lrcrcrcr}
(I) & -3x & + & 2y & - & 4z & = & 12 \\
(II) & & & y & + & 2z & = & 5 \\
(III) & 2x & + & 4y & + & 5z & = & 2
\end{array}$$

$$\begin{array}{lrcrcrcr}
2*(I) & -6x & + & 4y & - & 8z & = & 24 \\
3*(III) & 6x & + & 12y & + & 15z & = & 6 \\
\hline
(IV) & & & 16y & + & 7z & = & 30 \\
-16*(II) & & - & 16y & - & 32z & = & -80 \\
\hline
 & & & & - & 25z & = & -50 \\
 & & & & & z & = & 2 \\
 & & & & & y & = & 1 \\
 & & & & & x & = & -6
\end{array}$$

$(-6, 1, 2)$

Recap

- Eliminate one variable using one pair of equations.
- Eliminate the **same** variable from a different pair of equations.
- Solve the resulting 2 x 2 system of equations.
- Substitute to find the values of the other variables.