Solving 2 x 2 Systems of Linear Equations

Preliminaries and Objectives

Preliminaries

- Graphs of Lines
- Algebraic Skills
  - Distributive Law
  - Combining Like Terms
  - Solving Linear Equations

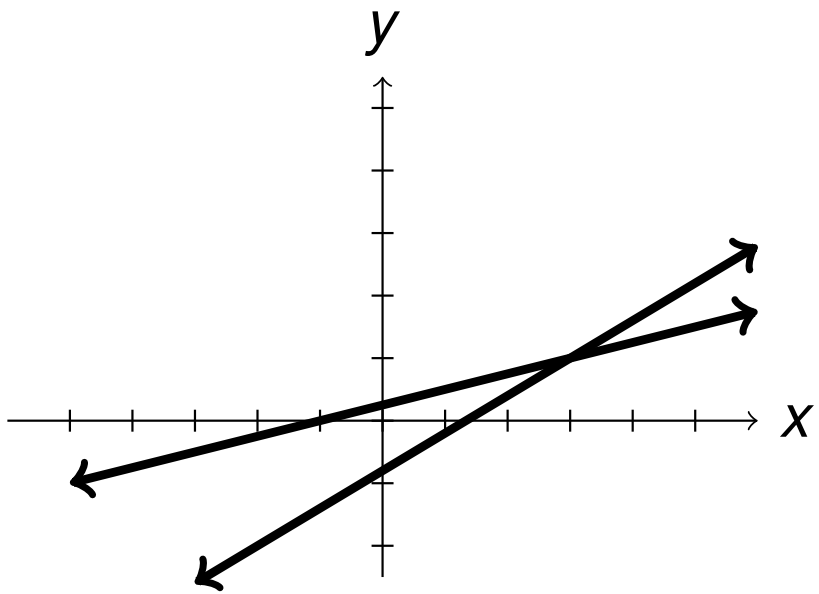
Objectives

- Determine if two lines intersect
- Find the intersection of two lines

Intersecting Lines

$$3x - 5y = 4$$

$$x = 4y - 1$$



Substitution Method

$$3x - 5y = 4$$
$$x = 4y - 1$$

$$3(4y - 1) - 5y = 4$$

$$12y - 3 - 5y = 4$$

$$7y = 7$$

$$y = 1$$

$$x = 4(1) - 1 = 3$$

Examples

$$2x - 3y = -8$$

$$8 = 4y$$

$$5x - y = 3$$

$$3x + 2y = 20$$

$$x = -1, \quad y = 2$$

$$x = 2, \quad y = 7$$

Elimination Method

$$\begin{aligned} 3x - 4y &= -3 \\ 5x + 2y &= 21 \quad \text{multiply by 2} \end{aligned}$$

$$\begin{array}{rcr} 3x - 4y & = & -3 \\ 10x + 4y & = & 42 \\ \hline 13x & = & 39 \end{array}$$

$$x = 3, \quad y = 3$$

Inconsistent Systems

$$2x - y = 7$$
$$-4x + 2y = 6$$

$$2x - y = 7$$
$$-2x + y = 3$$

$$0 = 10$$

No solutions

Dependent Systems

$$2x - y = 7$$
$$4x - 2y = 14$$

$$-4x + 2y = -14$$
$$4x - 2y = 14$$

$$0 = 0$$

Infinitely many solutions, the lines are the same line.

Solutions to a 2x2 System of Linear Equations

Solutions to Equations	Graph	Call the solutions
One x-value and one y-value	Lines Intersect	Consistent and Independent
False Statement	Parallel Lines	Inconsistent
True Statement	Overlapping Lines	Dependent