

Solving 3 x 3 Systems of Linear Equations



Preliminaries and Objectives

Preliminaries

- Solving a 2 x 2 system of linear equations
 - Substitution Method
 - Elimination Method

Objectives

- Find the solution to a system of 3 equations in three variables.

Solving 2 x 2 Systems of Equations - Elimination Method

- Multiply one or both equations by a constant so that one variable will cancel.
- Add equations together to get new equation with one variable.
- Solve for first variable.
- Substitute to find second variable.

Solving 3 x 3 Systems of Equations

- Pick two of the three equations and multiply one or both equations by a constant so that one variable will cancel.
- Add equations together to get new equation with two variables.
- Pick a different pair of equations and multiply one or both equations by a constant so that the **same** variable will cancel.
- Add equations together to get new equation with the **same** two variables.
- Solve the new 2 x 2 system

Example 1

$$\begin{array}{l} (I) \quad x + y + z = 0 \\ (II) \quad -2x + 2y - 4z = 12 \\ (III) \quad 2x - 3y - z = 7 \end{array}$$

$$\begin{array}{l} 2 * (I) \quad 2x + 2y + 2z = 0 \\ (II) \quad -2x + 2y - 4z = 12 \\ \hline (IV) \quad \quad \quad 4y - 2z = 12 \end{array}$$

$$(V) \quad -y - 5z = 19$$

Example 1

$$\begin{array}{l} (IV) \quad 4y - 2z = 12 \\ (V) \quad -y - 5z = 19 \end{array}$$

$$\begin{array}{l} (IV) \quad 4y - 2z = 12 \\ 4 * (V) \quad -4y - 20z = 76 \\ \hline \quad \quad \quad -22z = 88 \end{array}$$

$$\begin{array}{l} \quad \quad \quad z = -4 \\ (V) \quad -y + 20 = 19 \\ \quad \quad \quad y = 1 \\ (I) \quad x + y + z = 0 \\ (I) \quad x + 1 - 4 = 0 \\ \quad \quad \quad x = 3 \end{array}$$

$$(3, 1, -4)$$

Geometry of Linear Equations in 3-dimensional space

- Each linear equation is the equation of a plane.
- Two planes intersect in a line (usually).
- A line intersects the third equation (plane) at a point (usually).
- A false statement means two planes were parallel and there are no points of intersection of all three planes.
- A true statement means that there are infinitely many solutions, either because two planes were the same plane or that every pair of planes intersects at the same line.

Example 2

$$\begin{array}{l} (I) \quad -3x + 2y - 4z = 12 \\ (II) \quad \quad y + 2z = 5 \\ (III) \quad 2x + 4y + 5z = 2 \end{array}$$

$$\begin{array}{l} 2 * (I) \quad -6x + 4y - 8z = 24 \\ 3 * (II) \quad 6x + 12y + 15z = 6 \\ \hline (IV) \quad \quad \quad 16y + 7z = 30 \\ -16 * (II) \quad -16y - 32z = -80 \\ \hline \quad \quad \quad -25z = -50 \\ \quad \quad \quad z = 2 \\ \quad \quad \quad y = 1 \\ \quad \quad \quad x = -6 \end{array}$$

$$(-6, 1, 2)$$

Recap

- Eliminate one variable using one pair of equations.
- Eliminate the **same** variable from a different pair of equations.
- Solve the resulting 2×2 system of equations.
- Substitute to find the values of the other variables.