1. Solving 3x3 Systems of Equations

2. You should be familiar with solving a 2 x 2 system of linear equations including the substitution method and the elimination method.

   In this lesson, we will solve a system of three linear equations in three variables.

3. Recall the elimination method for solving a 2 x 2 system of equations. We first find a variable to eliminate and multiply one equation or both by constants. We then add the equations together to produce a new equation in one variable. We can then solve for that variable, and substitute the answer into a previous equation to find the other.

4. The method to solve a 3 x 3 system is similar. We attempt to reduce the problem to a 2 x 2 system by eliminating one variable. To do so, pick a pair of equations, and multiply by constants to cancel one variable. This will give one equation with two variables. Then pick another pair of equations and eliminate the **same** variable. We now have two equations in two variables. We can then solve this system.

5. (a) For example, we could solve this system in many ways. One approach is to eliminate z. We could add the first equation and third equation to get an equation involving x and y. We could then take four times the first equation with the second equation to get another equation involving x and y. We could then solve this system of equations.

      We could also eliminate y by combining 3 times the first equation with the third equation, and negative 2 times the first equation with the second equation.

      Here, lets eliminate x.

   (b) First, take 2 times the first equation and add the second equation. This results in the equation $4y - 2z = 12$. We now want to combine another pair of equations, eliminating x.

   (c) If we add the second and third equations together, we get another equation involving y and z.

   (d) We can now work with these two equations.

   (e) We can multiply equation V by 4.

   (f) Add it to equation IV

   (g) And solve for z

   (h) We can then plug in the value of -4 for z into any previous equation, like equation V,

   (i) to find y

   (j) and then plug y and z into equation I

   (k) to find x

   (l) The answer is the point $(3, 1, -4)$

6. Let's discuss the geometry briefly. The point is a point in 3-dimensional space. Each equation is the equation of a plane. When two equations are combined, we are finding the intersection of two planes, which is a line. When we intersect the line with a third plane, the intersection is a point. Usually. Just as happened with the 2 x 2 system, we can have parallel planes with no intersection, or overlapping solutions giving a full line or plane as the solution. In most cases, the intersection will be a point.

7. (a) Here is a second example. We have a head start since the second equation has already eliminated the x. We can combine the first and third equations to eliminate x again. Multiply equation I by 2 and equation III by 3
   (b) add them to eliminate x to get equation IV.
   (c) Take -16 times equation II and add it to equation IV
   (d) and solve for z. Plug 2 for z back into any equation, like equation II
   (e) to solve for y
   (f) Then plug z and y into any equation, like equation III, to solve for x.
   (g) The intersection point is $(-6, 1, 2)$

8. To recap: Eliminate one variable using one pair of equations. Then eliminate the same variable using a different pair of equations. You can know solve this 2 x 2 system of equations. Once you have found the answer for one variable, you can substitute to find the values for the other variables.