1. Solving 2 x 2 Systems of Linear Equations

2. You should be familiar with the graphs of lines and algebra skills like distributing and combining like terms to solve linear equations

   In this lesson, we will determine if two lines in the plane intersect, and if so, find the point of intersection.

3. We are interested in finding the point of intersection of two lines. One method is to graph the two lines and see where they intersect. This method gives a good graphical representation of what is going on, but it is often hard to tell exactly where the lines intersect.

4. We also have algebraic methods to find the answer. The first is the substitution method. If we can solve for one of the variables, we can substitute that value into the other equation. In this system, the second equation is already solved for x. We know $x = 4y-1$.

   We can replace the x in the first equation, with the expression that x equals. That is, in the first equation, we can substitute $4y - 1$ for x.

(b) We now have an equation containing only one variable, in this case, the variable y, which can now be solved, first distributing the 3

(c) Then combining like terms

(d) and finally dividing by 7. This is only part of the answer. Recall we are trying to find the ordered pair where the two lines intersect, we have only found the y-coordinate, we now need to find the x-coordinate. We do this by substituting the value of 1 for y in any equation. For example, we can substitute $y = 1$ into the second equation.

(e) From which we get $x = 3$

(f) The point $(3, 1)$ is the point of intersection of the two lines.

5. (a) Sometimes you may need to do a bit of work to use the substitution method. In the first example, you can divide y by 4 in the bottom equation and then substitute. In the second example you can move the y to the right side and the constant to the left in the first equation to solve for y. You may wish to pause the video to work out the solutions to these problems.

   (b) (*Answers provided*)

6. (a) A second method to solve systems of linear equations is called the elimination method. It relies on the fact that we can multiply both sides of an equation by the same constant to get a new equation. We can also add two equations together to get a new equation. The key is to choose constants in such a way that when we add the two equations together, one of the variables is eliminated.

   (b) In this case, if we multiply the bottom equation by 2,

   (c) then the $-4y$ will cancel the $4y$ and we will be left with an equation involving only x.

   (d) From here we can add the two equations together,

   (e) solve for x, and then substitute into any equation to find y. For example, substituting $x = 3$ into the first equation gives $9 - 4y = -3$, from which we find that $y = 3$.

7. (a) Two unusual things may happen in the solving process. In this example, we can divide the second equation by 2.

   (b) Now, when we add the equations together, both variables are eliminated

   (c) leaving a statement that is false. The reason this happened is that the lines were parallel and have no point of intersection. We call this system 'inconsistent'

8. (a) In this example, when we multiply the top line by -2

   (b) and add the equations together

   (c) We get the true statement $0 = 0$.

   The reason this happened is that the two equations represented the same line, so that every point (x, y) on the line is a solution. There are an infinite number of points that solve this equation.

   We call this system of equations 'dependent'

9. To recap. If in solving the system of equations, you find one x-value and y-value, the two lines intersected, and the system is called consistent and independent. The solution will be a single ordered pair. If in solving the system of equations you get a statement that is false, the two lines were parallel, and there is no solution. The system is called inconsistent. If in solving the system of equations you get a statement that is true, the two lines were overlapping, and the system is called dependent.