1. Factoring: Difference of Squares

2. You should be familiar with the Distributive Property and with Expanding Binomials. We will also refer to factoring integers and factoring processes in general. We will combine this factoring technique with other factoring techniques, in particular, we will give an example with a greatest common factor.

   In this lesson, we will factor binomials that are the Difference of Squares.

3. (a) Recall the FOIL process for expanding binomials. We begin by multiplying the first terms, then the outside terms, then the inside terms, and finally the last terms. This produces a four part answer.

   (b) Something special happens in this case. The outside product and the inside product are the same, except that one of them is positive and the other negative. When we combine like terms, these two terms cancel each other, and we are left with just the first term and the last term. This occurred because in the factored form, the two factors were identical, except that one contained a plus sign, and the other a minus sign. Thus, when we multiplied the first terms, we were multiplying x by itself, producing the square of x, and when we multiplied the last terms, we were multiplying 3 by itself, producing the square of 3, which is 9, with a negative sign since one term was positive and the other term was negative.

   (c) We now wish to reverse the process. This process only works when the first term is a perfect square, in this case, the variable x multiplied by itself, and the second term is a perfect square, in this case 3 times 3, with a minus sign in between. The factorization that works is to split apart the x^2 into two identical parts, x and x, then split apart the 9 into two identical multiplication parts 3×3. One of the terms contains a plus sign and the other a minus sign. Recall that when we expand this product, we get a four term expression with the middle terms canceling, leaving us with the Difference of Squares.

   This is the factored form of $x^2 - 9$

4. (a) Here is a second example. The initial term is the product of $2x$ with itself. The final term is 5-squared.

   (b) The factors are $(2x + 5)(2x - 5)$.

5. (a) Recall that the two terms must be separated by a minus sign for this technique to work. If we try to factor $x^2 + 4$, in order to produce the last term of positive 4, we would need to have both 2s positive, or both 2s negative. Either way, we would have a middle term.

   (b) $x^2 + 4$ will not factor.

6. (a) Here is a another example. Is the initial term x^4 the product of something with itself. The answer is yes, $x^4 = (x^2)(x^2)$, so we can factor $x^4 - 16$ as the difference of squares.

   (b) Note that second part can be factored further.

(c) Recall that when factoring integers, we may factor a large integer into smaller pieces, which then factor further using a factor tree. We carry downward any prime factors we have already discovered while continuing to factor the other parts.

(d) The same is true for polynomials, we may have steps when some parts are factored as completely as possible, while others can be factored further. We continue until all factors are as small as can be made and can't be made smaller. These are the irreducible factors we are looking for.

7. (a) Can $x^9 - 36$ be factored using the Difference of Squares?

   (b) If we try to split x^9 into pieces, we find that it can't be done evenly.

8. (a) Can $x^6 - 36$ be factored using the Difference of Squares?

   (b) In this case, $x^6 = (x^3)(x^3)$ and $36 = 6^2$, so this polynomial can be factored as a Difference of Squares.

9. (a) We can use more than one factoring technique while factoring. $3x^3 - 12x$ is not the Difference of Squares, however, the two terms do have a common factor.

   (b) We begin by factoring out the greatest common factor $3x$.

   (c) What remains is the difference of squares.

   (d) This is the factored form

10. (a) We can use more than one variable, as long as both terms are perfect squares.

    (b) This is the factored form

11. (a) Here are a few more for you to try. You may wish to pause the video now to work on these exercises.

    (b) .

12. To recap: The Difference of Squares technique can be applied when the expression to be factored has two terms, separated by a minus sign. Each term needs to be a perfect square so that it can be split evenly across the two factors.