1. Introduction to Analytic Geometry

2. You should be familiar with the Cartesian Coordinate System. It may be helpful to know the Slope Formula and the Pythagorean Theorem. In this lesson, we will write equations involving arbitrary points.

3. Many geometric ideas can be expressed using algebraic equations. The study of the relationship between geometry and algebra is called analytic geometry.

   One example is the slope formula defining the equation of a line. Recall that to calculate the slope between two points, the numerator is the difference in the y-coordinates, sometimes called the 'rise', and the denominator is the difference in the x-coordinates, the 'run'.

4. (a) What if we are interested in the slope between the point $(2, 1)$ and some other point, but in various applications, the other point keeps changing? We need a way to label this moving point.

   (b) We do so with variables. The unknown point is labelled as a variable point (x, y). What is the slope to the point $(2, 1)$? It depends on the location of the point (x, y). Different points will give different slopes.

   (c) What if we put a restriction on the point (x, y)? What geometric object do we get if we take all points such that the slope to the point $(2, 1)$ is $\frac{2}{3}$? It turns out that we get a line. Algebraically, what is the equation of the line? We can get the equation by writing the condition for the slope.

   This illustrates the main idea of analytic geometry: A condition that defines a geometric object can be represented by an algebraic equation involving an arbitrary point (x, y). The set of all points (x, y) that satisfy the equation is exactly the set that satisfies the geometric condition.

5. (a) Here is another example, the equation of a circle. We wish to find all points on a circle of radius 3, centered at $(-2, 4)$.

   (b) We label an arbitrary point (x, y) on the circle.

   (c) The Pythagorean Theorem gives the following equation, which is the equation of a circle.

6. To recap: we can convert a geometric description of an object to an algebraic expression by labeling an arbitrary point (x, y) on the geometric object, then writing an equation corresponding to the description of the geometric object.