

Solving Absolute Value Equations and Inequalities



Preliminaries and Objectives

Preliminaries

- Absolute Value
- Solving linear equations in one variable

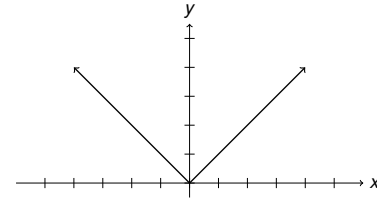
Objectives

- Solve equations involving absolute value
- Solve inequalities involving absolute value

Definition of Absolute Value

Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Example 1

$$\begin{aligned} |x| &= 3 \\ x &= 3 \text{ or } x = -3 \\ \{-3, 3\} \end{aligned}$$

Example 2

$$\begin{aligned} |x| &= -2 \\ \text{no solutions} \\ \emptyset \end{aligned}$$

Example 3

$$\begin{aligned} |x - 3| &= 2 \\ x - 3 &= 2 \text{ or } x - 3 = -2 \\ x &= 5 \text{ or } x = 1 \\ \{1, 5\} \end{aligned}$$

Example 4

$$\begin{aligned} |3x - 1| &= 5 \\ 3x - 1 &= 5 \text{ or } 3x - 1 = -5 \\ 3x &= 6 \text{ or } 3x = -4 \\ x &= 2 \text{ or } x = -\frac{4}{3} \\ \{-\frac{4}{3}, 2\} \end{aligned}$$

Example 5

$$\begin{aligned} |x| + 2 &= 4 \\ |x| &= 2 \\ x &= 2 \text{ or } x = -2 \\ \{-2, 2\} \end{aligned}$$

Example 6

$$\begin{aligned} 3|x - 2| + 1 &= 7 \\ 3|x - 2| &= 6 \\ |x - 2| &= 2 \\ x - 2 &= 2 \text{ or } x - 2 = -2 \\ x &= 4 \text{ or } x = 0 \\ \{0, 4\} \end{aligned}$$

Example 7

$$|x - 3| < 2$$

$$|x - 3| = 2$$

$$x - 3 = 2 \text{ or } x - 3 = -2$$

$$x = 5 \text{ or } x = 1$$



Example 8

$$|4x - 1| + 4 \geq 7$$

$$|4x - 1| \geq 3$$

$$4x - 1 = 3 \text{ or } 4x - 1 = -3$$

$$4x = 4 \text{ or } 4x = -2$$

$$x = 1 \text{ or } x = -\frac{1}{2}$$

