

Solving Absolute Value Equations and Inequalities



Preliminaries and Objectives

Preliminaries

- Absolute Value
- Solving linear equations in one variable

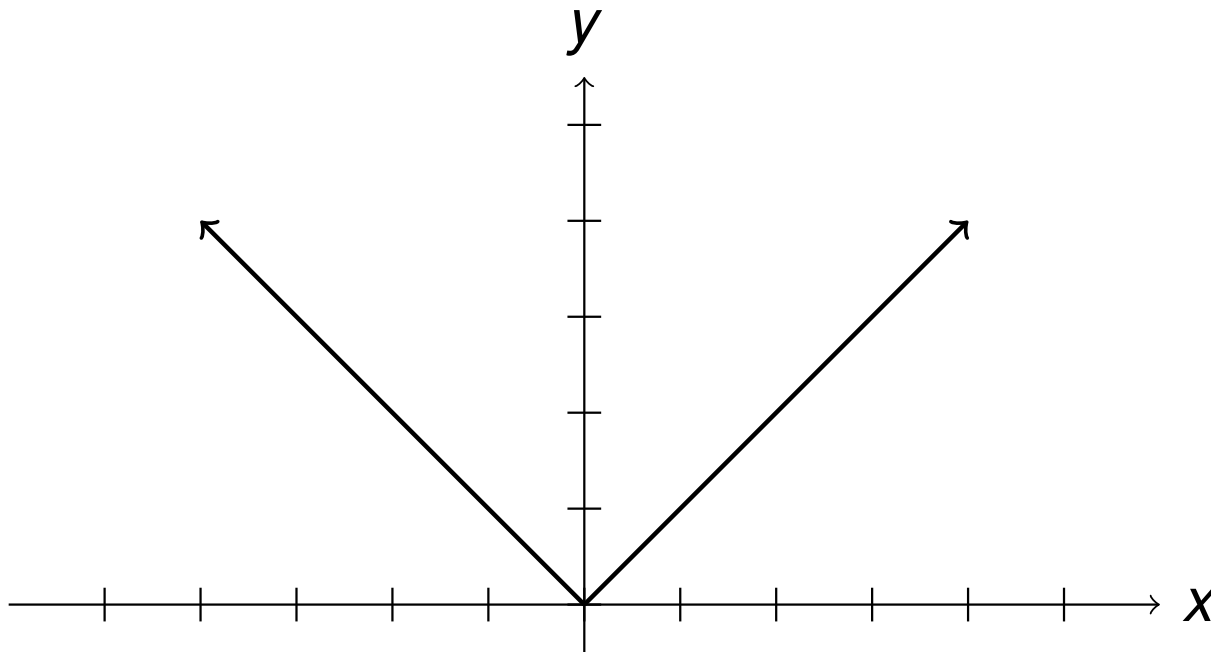
Objectives

- Solve equations involving absolute value
- Solve inequalities involving absolute value

Definition of Absolute Value

Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Example 1

$$|x| = 3$$

$$x = 3 \text{ or } x = -3$$

$$\{-3, 3\}$$

Example 2

$$|x| = -2$$

no solutions

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Example 3

$$|x - 3| = 2$$

$$x - 3 = 2 \text{ or } x - 3 = -2$$

$$x = 5 \text{ or } x = 1$$

$$\{1, 5\}$$

Example 4

$$|3x - 1| = 5$$

$$3x - 1 = 5 \text{ or } 3x - 1 = -5$$

$$3x = 6 \text{ or } 3x = -4$$

$$x = 2 \text{ or } x = -\frac{4}{3}$$

$$\left\{-\frac{4}{3}, 2\right\}$$

Example 5

$$|x| + 2 = 4$$

$$|x| = 2$$

$$x = 2 \text{ or } x = -2$$

$$\{-2, 2\}$$

Example 6

$$3|x - 2| + 1 = 7$$

$$3|x - 2| = 6$$

$$|x - 2| = 2$$

$$x - 2 = 2 \text{ or } x - 2 = -2$$

$$x = 4 \text{ or } x = 0$$

$$\{0, 4\}$$

Example 7

$$|x - 3| < 2$$

$$|x - 3| = 2$$

$$x - 3 = 2 \text{ or } x - 3 = -2$$

$$x = 5 \text{ or } x = 1$$



Example 8

$$|4x - 1| + 4 \geq 7$$

$$|4x - 1| \geq 3$$

$$4x - 1 = 3 \text{ or } 4x - 1 = -3$$

$$4x = 4 \text{ or } 4x = -2$$

$$x = 1 \text{ or } x = -\frac{1}{2}$$

