1. The Absolute Value Function

2. It may be helpful to be familiar with equations of lines and with piecewise defined functions. In this lesson, we will define the absolute value function.

3. One standard way to set up a navigational system is the Cartesian Coordinate System, in which the x-coordinate tells us how far to go left and right and the y-coordinate tells us how far to go up and down.

4. (a) In other navigational contexts, traveling from Minneapolis to Chicago, for example, we may be given information differently.

   (b) The first piece of information is the distance; we need to know how far to go.

   (c) We also need to know the compass heading, that is, we need to know which direction to head.

5. (a) On the number line, things are simplified somewhat. There are only two directions we can head from zero, either left or right. We know the direction based on the plus or minus sign. It is the sign that tells us the direction. The distance is the 'size' of the number, called the absolute value.

   (b) There are two points that are a distance of 3 from the origin, $+3$ and -3. The $\pm$ sign tells us whether to go left or right, the 3 tells us how far to go. The 3 is the absolute value. In other words, the absolute value is the distance from zero. The plus or minus is the direction.

6. (a) The formal definition is a two-part, piecewise definition. The absolute value of x is x, if x is positive, and the opposite, if x is negative. It may look as though the output on the bottom line is negative, because of the negative sign, but recall that $x < 0$, so $-x$ is a double negative, and therefore a positive number. The absolute value is always a positive distance. If the thing you are taking the absolute value of is negative, merely throw away the negative sign, and keep the positive part.

   (b) Here is a graph of the absolute value function. To the right of the y-axis, where x is positive, the graph is the line $y = x$. In other words, a line through the origin with a slope of 1. When x is negative, the y-value is positive, that is, the y-value has the opposite sign as the x-value, so the graph is the line $y = -x$.