1. Solving Absolute Value Equations and Inequalities

2. You should be familiar with the absolute value function and with solving linear equations in one variable. In this lesson, we will solve equations involving the absolute value function and inequalities involving the absolute value function.

3. Recall the absolute value function. If the thing we are taking the absolute value of is positive, we leave it unchanged. If the thing we are taking the absolute value of is negative, we change its sign.

4. When solving equations, we are therefore given two possibilities for the absolute value. In this case, perhaps $x$ is positive 3, or perhaps we removed the negative sign from -3. In set notation, the solution is the two numbers -3 and 3.

5. Recall that the absolute value will always be positive. Here, we are asked to find when an absolute value is negative, which will never happen. The solution is the emptyset.

6. Once again, the thing we are taking the absolute value of, in this case $x - 3$, could either be 2 or -2. We need to solve for both possibilities. We begin by setting the thing inside the absolute value equal to both 2 and -2, and solving these equations separately by adding 3 to both sides.

7. This problem is similar. The thing inside the absolute value can be either 5 or -5. This leads to two equations. We add one to both sides, then divide by 3 to get the solutions.

8. In this case, there is some manipulation to be done first. There are no procedures for moving numbers into and out of the absolute value, only the definition of the absolute value, so we must first isolate the absolute value. We do this by subtracting 2 from both sides.

9. Here, we first subtract 1, then divide by 3. This isolates the absolute value. We then solve the absolute value problem in the standard way.

10. For an inequality, it is best to think of absolute value as a distance. In this case the distance is less than 2. That means we will be looking for the points that are nearby. It is easier to solve the related equation first. If we solve $|x-3| = 2$, we find that the two points which have a distance of 2 are located at 1 and 5. Since we want the points that are a distance less than 2 away from 3, we take the points between 1 and 5.

11. This example is similar. First, isolate the absolute value, and solve the equation. Thinking again of absolute value as a distance, we are looking for points whose distance is greater than some value, and therefore are far away, so we need the points outside of the interval from -1/2 to 1.