Graphing Rational Functions

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Preliminaries and Objectives

Preliminaries

- Intercepts
- Factoring Polynomials
- Graphing Polynomials
- Long Division of Polynomials

Objectives

- Graph Rational Functions

Example 1

$$f(x) = \frac{x^5 + 5x^4 - 5x^3 - 45x^2 + 108}{x^3 - x^2 - x + 1} = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$

Keys to Graphing

- y-intercept
- x-intercepts
- vertical asymptotes
- end behavior

Example 1

Step 1: Find the y-intercept

$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$

$$\begin{aligned} f(0) &= \frac{(0+3)^3(0-2)^2}{(0+1)(0-1)^2} \\ &= 108 \end{aligned}$$

Example 1

Step 2: Find the x-intercepts

$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$

$$\begin{array}{ccc} (x+3)=0 & \text{or} & (x-2)=0 \\ x=-3 & \text{or} & x=2 \end{array}$$

x-intercepts at -3 and 2

Example 1

Step 3: Find asymptotes

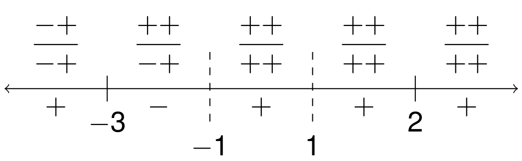
$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$

$f(x)$ is undefined when $x = -1$ and when $x = 1$

Example 1

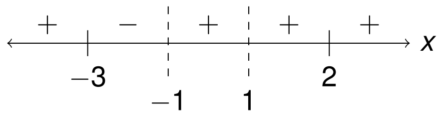
Step 4: Analyze intervals

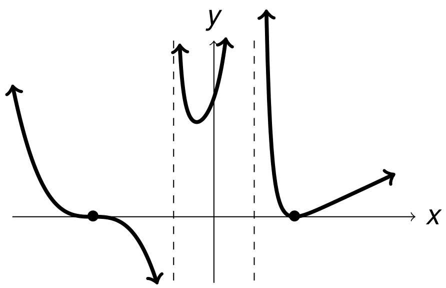
$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$



Example 1

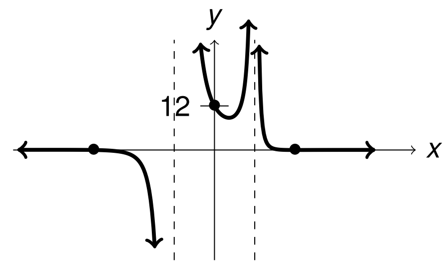
$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$

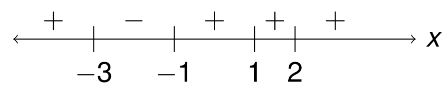




Example 2

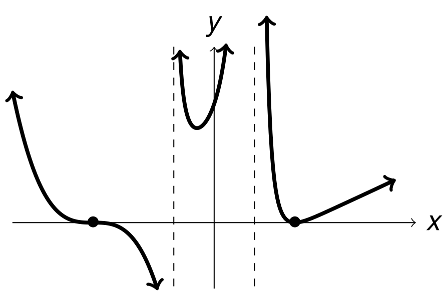
$$f(x) = \frac{x^3 - x^2 - 8x + 12}{x^4 + 2x^3 - 2x - 1}$$





Example 1

$$f(x) = \frac{x^5 + 5x^4 - 5x^3 - 45x^2 + 108}{x^3 - x^2 - x + 1}$$



End Behavior

$$f(x) = \frac{x^2}{x^4 - 2x^2 + 1}$$

- If the degree of the numerator is less than the degree of the denominator, then

$$\lim_{x\to\infty} f(x) = 0 \text{ and } \lim_{x\to-\infty} f(x) = 0$$

End Behavior

$$f(x) = \frac{2x^2 - 18}{x^2 + 1}$$

- If the degree of the numerator is the same as the degree of the denominator, then long division will produce a constant and a remainder. In this case, the constant is a horizontal asymptote.

End Behavior

$$f(x) = \frac{3x^3 + x^2 + 3x + 6}{x^2 + 1}$$

- If the degree of the numerator is larger than the degree of the denominator, then long division will produce two terms and a remainder. In this case the graph approaches an oblique asymptote.

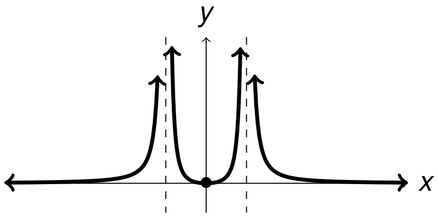
End Behavior

$$f(x) = \frac{3x^4 + x^3 - 3x^2 - x + 5}{x^2 - 1}$$

- If the degree of the numerator is at least two greater than the degree of the denominator, then the graph resembles x^2, x^3, x^4,...

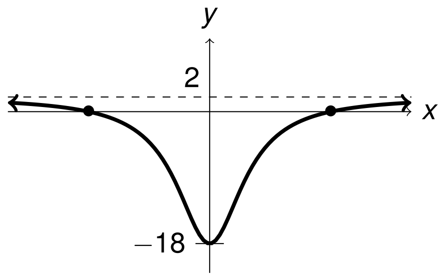
End Behavior

$$f(x) = \frac{x^2}{x^4 - 2x^2 + 1} = \frac{x^2}{(x-1)^2(x+1)^2}$$



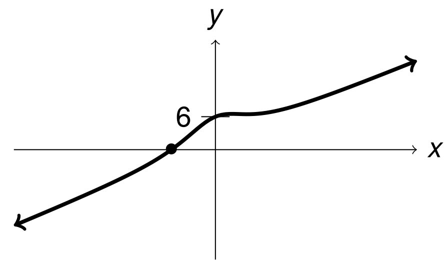
End Behavior

$$f(x) = \frac{2x^2 - 18}{x^2 + 1} = 2 - \frac{20}{x^2 + 1}$$



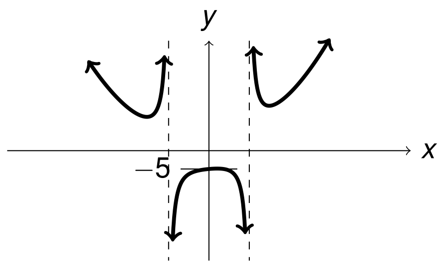
End Behavior

$$f(x) = \frac{3x^3 + x^2 + 3x + 6}{x^2 + 1} = 3x + 1 + \frac{5}{x^2 + 1}$$



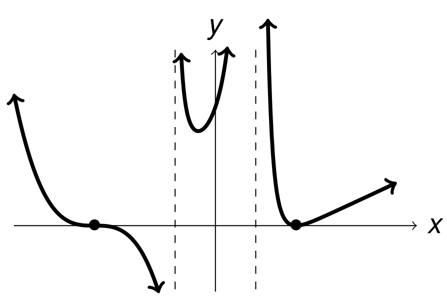
End Behavior

$$f(x) = \frac{3x^4 + x^3 - 3x^2 - x + 5}{x^2 - 1} = 3x^2 + x + \frac{5}{x^2 - 1}$$



End Behavior

$$f(x) = \frac{x^5 + 5x^4 - 5x^3 - 45x^2 + 108}{x^3 - x^2 - x + 1}$$



Recap

- Find y-intercept
- Find x-intercepts
- Find vertical asymptotoes
- Determine end behavior
- Sketch the graph