The Composition of Functions

University of Minnesota

Preliminaries and Objectives

Preliminaries

- Functions
- Function Notation

Objectives

- Define the composition of functions

University of Minnesota The Composition of Functions

Functions and Notation

$$f(x) = x^2$$

$$f(t) = t^2$$

$$f(-3) = 9$$

$$f\Big(g(x)\Big) = \Big[g(x)\Big]^2$$

University of Minnesota The Composition of Functions

Composition of Functions

$$f(x) = x^2 \qquad g(x) = x - 3$$

$$f\Big(\bullet\Big) = \bullet^2$$

$$f\Big(g(x)\Big) = (x-3)^2$$

University of Minnesota The Composition of Functions

Composition of Functions

$f(x) = x^2$ $\qquad$ $g(\bullet) = \bullet - 3$

$$f(g(x)) = (x-3)^2$$

$$g(f(x)) = x^2 - 3$$

Example 2

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = 3x$

Find $f(g(x))$ and $g(f(x))$

$$f(g(x)) = \sqrt{3x}$$

$$g(f(x)) = 3\sqrt{x}$$

Example 3

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = x - 6$ $\qquad$ $h(x) = 3x$

Find $f(g(h(x)))$

$$\begin{aligned} h(x) &= 3x \\ g(h(x)) &= g(3x) = 3x - 6 \\ f(g(h(x))) &= f(3x-6) \\ &= \sqrt{3x-6} \end{aligned}$$

Recap

To find $f(g(x))$, use the output of $g(x)$ as the input to $f(x)$.