Polynomial and Rational Inequalities

University of Minnesota

Preliminaries and Objectives

Preliminaries

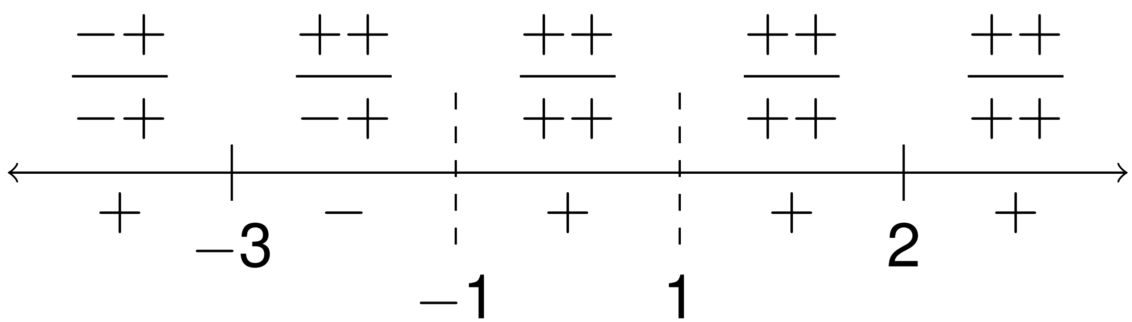
- Graphing Polynomials
- Graphing Rational Functions
- Interval Notation

Objectives

- Solve Polynomial Inequalities
- Solve Rational Inequalities

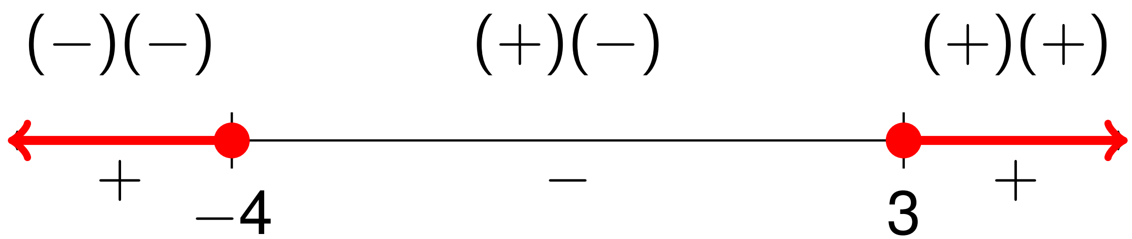
Example 1

$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2}$$



Example 2

$$(x+4)(x-3) \geq 0$$

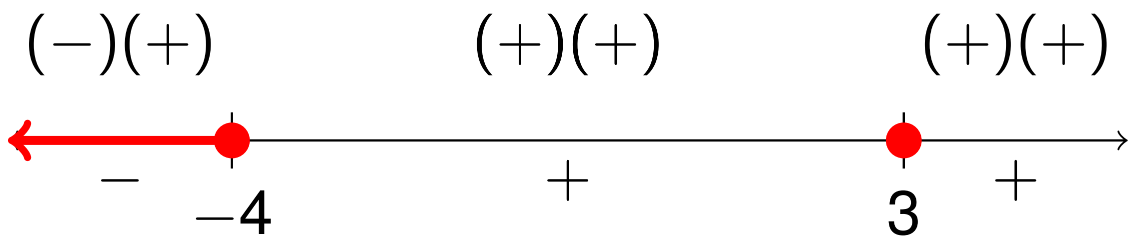


The set of all values x for which $(x+4)(x-3) \geq 0$ is

$$(-\infty, -4] \cup [3, \infty)$$

Example 3

$$(x+4)^3(x-3)^2 \leq 0$$



The set of all values x for which $(x+4)^3(x-3)^2 \leq 0$ is

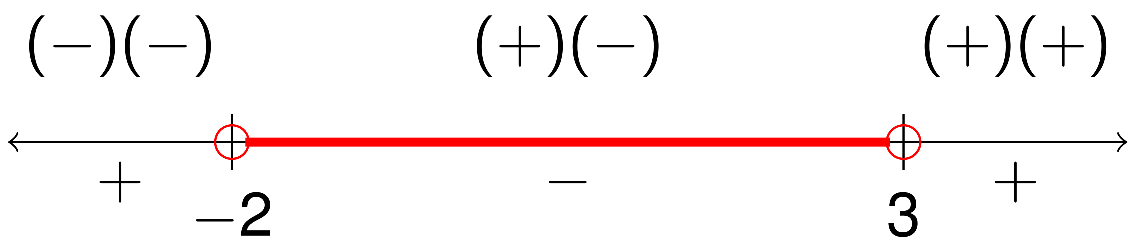
$$(-\infty, -4] \cup \{3\}$$

Example 4

$$x^2 < x + 6$$

$$x^2 - x - 6 < 0$$

$$(x + 2)(x - 3) < 0$$

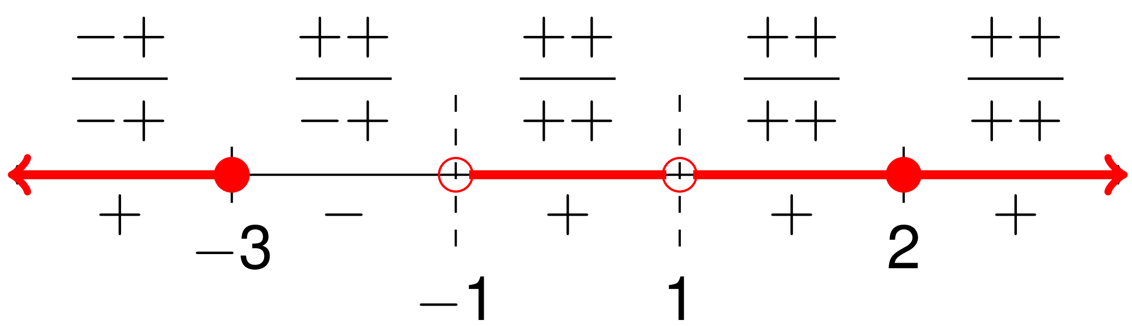


The set of all values x for which $x^2 - x - 6 < 0$ is

$$(-2, 3)$$

Example 1

$$f(x) = \frac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2} \geq 0$$



The set of all values x for which $\dfrac{(x+3)^3(x-2)^2}{(x+1)(x-1)^2} \geq 0$ is

$$(-\infty, -3] \cup (-1, 1) \cup (1, \infty)$$

Recap

- Set one side of the inequality equal to zero
- Factor
- Divide the number line by placing the x-intercepts and asymptotes
- Analyze the factors to determine on which intervals the function is positive/negative
- For $\leq$ and $\geq$, include the x-intercepts as the endpoints of the intervals
- Never include the x-values associated with asymptotes, as the function is undefined at these points.