## 1. One-to-One Functions

- 2. You should be familiar with functions, function notation and graphs of functions. You should also be familiar with inverse functions. In this lesson, we will define what it means for a function to be one-to-one.
- 3. Often we are asked to undo a process, for example, subtraction undoes addition. This is the concept of an inverse function. Occasionally, there is some difficulty in defining the reverse process. For example, for the function  $f(x) = x^2$ , when we input 2, we get the output 4. When we input -2, we also get the output 4. When we try to reverse this operation with the square root, we are not sure whether we should pick 2 or -2 to associate with 4. The problem is that there were two inputs corresponding to the output 4. We avoid this problem with functions that have a special property, where each output corresponds to only one input, called **one-to-one** functions.
- 4. A one-to-one function has just one input for each output. This allows us to define the inverse function. If each y output of the original function corresponds to only one x input, then each y input to the inverse function will correspond to a single x output.

The test to see if a function is one-to-one is called the **horizontal line test**. If a vertical line crosses the graph more than once, the graph is not a function, since a single x-input is associated with more than one output. If a horizontal line crosses the graph more than once, the graph is not one-to-one, since a single y-output is associated with more than one input.

- 5. A line with positive slope is one-to-one, since each y output will intersect the line exactly once.
- 6.  $y = x^2$  is not one-to-one, since the y output of 4 corresponds to both the x-values 2 and -2.
- 7. Any continuous one-to-one function will either be strictly increasing or strictly decreasing. Once a function begins to go up, it can't turn around and go down and still be one-to-one.
- 8. The exponential function is increasing, the larger the exponent, the larger the value. The exponential function is one-to-one. It's inverse is called the logarithm function.
- 9. The function  $f(x) = \sin(x)$  is not one-to-one. For each y-value, there are many angles x associated with it.
- 10. To recap: If any horizontal line crosses the graph more than once, a function is not one-to-one, and we will have a problem finding an inverse function. If each horizontal line crosses a graph only once, then the function is a one-to-one function.