1. Inverse Functions - Part II

2. You should be familiar with inverse functions and one-to-one functions. In this lesson, we will define inverses for some common functions.

3. This is the graph of the exponential function, a continuous, increasing, one-to-one function. It is a function, since each x-value produces only one y-value, that is, it passes the vertical line test. It is one-to-one, since each y-value corresponds to only one x-value, that is, it passes the horizontal line test. We can easily find the complete inverse function by interchanging x and y. The inverse function is called the logarithm function.

4. (a) Certain functions are not one-to-one, so we must restrict the domain to a portion of the graph that is one-to-one to find an inverse. For example, the function $y = x^2$ is not one-to-one, each y-value corresponds to two x-values, one positive, one negative. If we interchange x and y, we get a graph that is not a function.

   (b) When defining the function $y = \sqrt{x}$ to be the inverse of squaring, we need to make a decision as to which branch of the graph to use. For the square root function, we choose the positive answer.

   (c) We can achieve the same effect by returning to the function $y = x^2$ and choosing a branch that is one-to-one.

5. (a) Two other functions that are not one-to-one are $y = \sin x$ and $y = \cos x$. Here is a graph of $y = \sin x$. We use the same technique, choosing a portion of the graph that is one-to-one.

   (b) $y = \sin x$ has its minimum y-value at -1 and maximum at 1.

   (c) We restrict the domain to a portion of the graph that is one-to-one.

6. (Animation) If we interchange x and y on this portion of the graph, we get an inverse function for $y = \sin x$. Let's see this again using the entire function. If we interchange x and y, we will get a graph that is not a function. We need to choose a portion of the graph that is a function to serve as the inverse.

7. To recap: If a function is one-to-one, we can find the inverse by interchanging x and y. If a function is not one-to-one, we need to restrict the domain to a portion of the graph that is one-to-one to find an inverse.