1. The Composition of Functions

2. You should be familiar with functions and function notation. In this lesson, we will define the composition of two functions.

3. (a) Recall that a function transforms an input to an output, for example, the function f squares the input to produce the output.
   (b) It does not matter what variable is used for the input, the function f squares the input.
   (c) The input may be a specific number, like -3. The function will square -3 to give the output of 9.
   (d) Sometimes we use an entire function as the input. We could use the output of the g function, $g(x)$ as the input to the f function.

4. (Animation) In essence, what we have done is split a complicated function into smaller steps. If g subtracts 3 and f squares, then the composition first subtracts 3, then squares. For example, if 7 goes into the g function, 4 will come out, which then gets squared in f to produce the answer 16. If we begin with 0 as the input to g, we get 9 as our final answer. In general, when an arbitrary input x goes into g, the output is $(x-3)$. The $(x-3)$ is then the input to f, which gets squared to produce the answer $(x-3)^2$.

5. (a) Here is the previous example, presented formally. Recall that the input variable to a function is merely a symbol to help identify where the input is located in the function.
   (b) We could use a different variable,
   (c) or just indicate it with a spot,
   (d) or use a different color.
   (e) The input to the function f is the output of the function g, which is $x-3$, so we replace the input to the f function with $x-3$.

6. (a) Does the order of the two functions matter? In general, the answer is yes.
   (b) When we use x^2 as the input to the g function,
   (c) the x^2 replaces the red dot, and we get a different answer, x^2-3.

7. (a) Here is another example. To find the first answer, use the output of the g function as the input to the f function, that is, plug $3x$ into the f function.
   (b) The f function takes the square root of the input, which in this case, is the g function, shown in red, so that we are taking the square root of the g function.
   (c) To find the second answer, use the output of the f function, that is, use $\sqrt{x}$, as the input to the g function. Again the input function is shown in red, and in this case, the g function multiplies the input by 3.

8. We can compose as many functions as we like. $f(g(h(x)))$ will be the function that multiplies by 3, then subtracts 6, then takes the square root.

9. To recap: to compose two or more functions, use the output of the inner function as the input of the outer function.