Exponential Growth and Geometric Sequences

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Preliminaries and Objectives

Preliminaries
- Sequences
- Linear Growth
- Arithmetic Sequences

Objectives
- Find values of a geometric sequence defined explicitly
- Find values of a geometric sequence defined recursively
- Find a recursive formula for a geometric sequence
- Find an explicit formula for a geometric sequence

Recursive Definition

Let $g_1 = 5$ and $g_{n+1} = (10)(g_n)$

5 50 500 5000 ___ ___ ___

Explicit Definition

Let $g_n = (16)\left(\frac{1}{2}\right)^n$

16 8 4 2 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$

Finding the formulas

Given the geometric sequence

3 12 48 192 768 3072

find a recursive formula and an explicit formula.

Recursive Formula:
$g_0 = 3$, $g_{n+1} = (4)(g_n)$

Explicit Formula:
$g_n = 3(4^n)$

Example 1

If a geometric sequence contains the terms $g_3 = 2$ and $g_7 = 162$, find a recursive formula and an explicit formula for g_n.

$\frac{2}{27}$ $\frac{2}{9}$ $\frac{2}{3}$ 2 6 18 54 162 ___ ___ ___ ___ ___ ...

g_1 g_3 g_7

$r =$ 4th root of $\frac{162}{2} =$ 4th root of $81 = 3$

Recursive definition: $g_1 = \frac{2}{9}$; $g_{n+1} = (3)(g_n)$

Explicit Definition: $g_n = \frac{2}{27}(3^n)$

Example 2

You put a $500 purchase on your credit card. Each month, interest is compounded until at the end of 9 months, you owe $571.70. What is the interest rate?

$g_0 = 500$, $g_9 = 571.70$

$g_9 = g_0(r^9) \Rightarrow r^9 = \frac{571.70}{500} = 1.1434 \Rightarrow r = 1.015$

The interest rate is 1.5% per month or 18% annually.

Recap

- Recursive definition: State the value of g_0 and the recursion $g_{n+1} = (r)(g_n)$
- Explicit definition: $g_n = (g_0)r^n$