Linear Growth and Arithmetic Sequences

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Preliminaries and Objectives

Preliminaries
- Represent data
- Equations of Lines
- Sequences

Objectives
- Find values of an arithmetic sequences defined explicitly
- Find values of an arithmetic sequences defined recursively
- Find a recursive formula for an arithmetic sequence
- Find an explicit formula for an arithmetic sequence

Constant Growth

Day	Boxes
0	37
1	40
2	43
3	46
4	49
⋮	⋮
9	?
⋮	⋮
?	100

- How many boxes are in the warehouse after Day 9?
- After how many days will the warehouse have 100 boxes?
- Can we find a general formula for the number of boxes after Day n?

$a_n =$

Constant Growth

Day	Boxes
0	37
1	40
2	43
3	46
4	49
⋮	⋮
9	?
⋮	⋮
?	100

$a_n =$ number of boxes after Day n

$a_1 = 40$, $a_2 = 43$, $a_3 = 46$, $a_4 = 49$

$a_{next} = a_{prev} + 3$

$a_{n+1} = a_n + 3$, $a_1 = 40$, $a_0 = 37$

$a_{next} = a_{prev} + d$

$a_n = 3n + 37$

$y = mx + b$

Constant Growth

Day	Boxes
0	37
1	40
2	43
3	46
4	49
⋮	⋮
9	?
⋮	⋮
?	100

How many boxes are in the warehouse after Day 9?

$a_9 = 3(9) + 37 = 64$

Constant Growth

Day	Boxes
0	37
1	40
2	43
3	46
4	49
⋮	⋮
9	?
⋮	⋮
?	100

After how many days will the warehouse have 100 boxes?

Find n, if $a_n = 100$

$100 = 3(n) + 37$

$100 - 37 = 3n$

$63 = 3n$

$\frac{63}{3} = n$

$21 = n$

Recursive Definition

Let $a_1 = 9$ and $a_{n+1} = a_n + 2$

9 11 13 15 17 19 21

Explicit Definition

Let $a_n = -5n + 13$

8 3 −2 −7 −12 −17 −22

Finding the formulas

Given the arithmetic sequence

57 54 51 48 45 42 39

find a recursive formula and an explicit formula.

Recursive Formula:
$a_0 = 57$, $a_{n+1} = a_n - 3$

Explicit Formula:
$a_n = -3n + 57$

Example

If an arithmetic sequence contains the terms $a_7 = 52$ and $a_{12} = 82$, find a recursive formula and an explicit formula for a_n.

$$\underset{}{10}\ \underset{a_1}{16}\ \underset{a_2}{22}\ \underset{a_3}{28}\ \underset{a_4}{34}\ \underset{a_5}{40}\ \underset{a_6}{46}\ \underset{a_7}{52}\ \underset{a_8}{58}\ \underset{a_9}{64}\ \underset{a_{10}}{70}\ \underset{a_{11}}{76}\ \underset{a_{12}}{82}\ \cdots$$

$$d = \frac{82 - 52}{12 - 7} = 6$$

Recursive definition: $a_1 = 16$; $a_{n+1} = a_n + 6$

Explicit Definition: $a_n = 6n + 10$

Recap

- Recursive definition: State the value of a_1 and the recursion $a_{n+1} = a_n + d$
- Explicit definition: $a_n = d(n) + a_0$