Examples Using the Properties of Logarithms

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries
- Laws of Logarithms
- Laws of Exponents

Objectives
- Simplify expressions using logarithms
- Solve equations involving logarithms

Laws of Logarithms

$$b^m = x \qquad\qquad \log_b x = m$$
$$b^n = y \qquad\qquad \log_b y = n$$

$$\log_b 1 = 0 \qquad\qquad \log_b \frac{1}{x} = -\log_b x$$

$$\log_b b = 1 \qquad\qquad \log_b xy = \log_b x + \log_b y$$

$$\log_b b^m = m \qquad\qquad \log_b \frac{x}{y} = \log_b x - \log_b y$$

$$b^{\log_b x} = x \qquad\qquad \log_b x^n = n \cdot \log_b x$$

Example 4

Write $2\log_{10} 3$ as a single logarithm

$$2\log_{10} 3 = \log_{10} 3^2 = \log_{10} 9$$

Example 5

Write as a single logarithm

$$\log_{10} 7 + \log_{10} 4 = \log_{10}(7)(4) = \log_{10} 28$$

Example 6

$$3\log_{10} 6 - 2\log_{10} 5$$
$$= \log_{10} 6^3 - \log_{10} 5^2$$
$$= \log_{10} 216 - \log_{10} 25$$
$$= \log_{10} \frac{216}{25}$$
$$= \log_{10} 8.64$$

Example 7

$$3^{\log_3 10 - \log_3 7}$$
$$= 3^{\log_3 \frac{10}{7}}$$
$$= \frac{10}{7}$$

Example 8

Solve for x

$$\log_4 x = 2$$

Ans: $\quad 4^2 = x$
$\qquad\quad x = 16$

Example 9

Solve for x

$$\log_{10}(3x + 1) = 1$$

Ans: $\quad 10^1 = 3x + 1$
$\qquad\quad x = 3$

Example 10

Solve for x

$$\log_4(x + 2) = \log_4 8$$

Ans: $x + 2 = 8$
       $x = 6$

Example 11

Solve for x

$$\log_{10} 2x - \log_{10}(x - 3) = 1$$

Ans: $\log_{10} \dfrac{2x}{x - 3} = 1$

$$10^1 = \frac{2x}{x - 3}$$

$$10(x - 3) = 2x$$

$$10x - 30 = 2x$$

$$8x = 30$$

$$x = \frac{30}{8}$$