Examples Using the Properties of Logarithms

Preliminaries and Objectives

Preliminaries
- Laws of Logarithms
- Laws of Exponents

Objectives
- Simplify expressions using logarithms
- Solve equations involving logarithms

Laws of Logarithms

\[
\begin{align*}
b^m &= x \\
b^n &= y \\
\log_b 1 &= 0 \\
\log_b b &= 1 \\
\log_b b^m &= m \\
\log_b x &= x \\
\log_b xy &= \log_b x + \log_b y \\
\log_b \left(\frac{x}{y}\right) &= \log_b x - \log_b y \\
y^{\log_b x} &= x
\end{align*}
\]

Example 4

Write \(2 \log_{10} 3\) as a single logarithm

\[
2 \log_{10} 3 = \log_{10} 3^2 = \log_{10} 9
\]

Example 5

Write as a single logarithm

\[
\log_{10} 7 + \log_{10} 4 = \log_{10} (7 \cdot 4) = \log_{10} 28
\]

Example 6

\[
3 \log_{10} 6 - 2 \log_{10} 5
\]

\[
= \log_{10} 6^3 - \log_{10} 5^2
\]

\[
= \log_{10} 216 - \log_{10} 25
\]

\[
= \log_{10} \frac{216}{25}
\]

\[
= \log_{10} 8.64
\]

Example 7

\[
3 \log_{10} 10 - \log_{10} 7
\]

\[
= 3 \cdot 10 - \log_{10} 7
\]

\[
= 10 - \frac{\log_{10} 7}{7}
\]

Example 8

Solve for \(x\)

\[
\log_4 x = 2
\]

\[
\text{Ans: } 4^2 = x
\]

\[
x = 16
\]

Example 9

Solve for \(x\)

\[
\log_{10} (3x + 1) = 1
\]

\[
\text{Ans: } 10^1 = 3x + 1
\]

\[
x = 3
\]
Example 10

Solve for $x$

\[ \log_4(x + 2) = \log_4 8 \]

Ans: \[ x + 2 = 8 \]
\[ x = 6 \]

Example 11

Solve for $x$

\[ \log_{10} 2x - \log_{10}(x - 3) = 1 \]

Ans:
\[ \log_{10} \frac{2x}{x - 3} = 1 \]
\[ 10^1 = \frac{2x}{x - 3} \]
\[ 10(x - 3) = 2x \]
\[ 10x - 30 = 2x \]
\[ 8x = 30 \]
\[ x = \frac{30}{8} \]