Newton’s Law of Cooling

Preliminaries and Objectives

Preliminaries:
• Standard model for exponential growth and decay
  \[ A(t) = A_0 e^{rt} \]
• Conversion between logarithmic form and exponential form

Objectives:
• Solve problems using Newton’s Law of Cooling

Cooling to Room Temperature

Example

A cup of coffee is heated to 80°C. It is placed in a room whose temperature is 20°C. After 2 minutes, the temperature has decreased to 60°C. At what time will the coffee have cooled to 40°C?

\[
\begin{array}{c|c|c}
\text{Time} & \text{Temp above r.t.} & \text{Temp above r.t.} \\
0 & 80 - 20 = 60 & 40 = 60e^{-2k} \\
2 & 60 - 20 = 40 & 20 = 60e^{-0.20273t} \\
? & 40 - 20 = 20 & \\
\end{array}
\]

Recap

Newton’s Law of Cooling

\[ T^*(t) = T_0^* e^{-kt} \]

All temperatures are measured as temperature above room temperature.

Details of Example

\[ T^*(t) = T_0^* e^{-kt} \]
\[ \Rightarrow 40 = 60e^{-2k} \]
\[ \Rightarrow \frac{40}{60} = e^{-2k} \]
\[ \Rightarrow \ln \frac{40}{60} = -2k \]
\[ \Rightarrow \ln \frac{4}{3} = 2k \]
\[ \Rightarrow k = \frac{\ln \frac{4}{3}}{2} \]
\[ \Rightarrow k \approx 0.20273 \]

Details of Example

\[ T^*(t) = T_0^* e^{-kt} \]
\[ \Rightarrow 20 = 60e^{-0.20273t} \]
\[ \Rightarrow \frac{20}{60} = e^{-0.20273t} \]
\[ \Rightarrow \ln \frac{2}{3} = -0.20273t \]
\[ \Rightarrow t = \frac{\ln \frac{2}{3}}{-0.20273} \]
\[ \Rightarrow t \approx 5.42 \]