Newton's Law of Cooling

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries:

- Standard model for exponential growth and decay

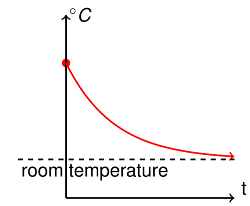
$$A(t) = A_0 e^{rt}$$

- Conversion between logaritmic form and exponential form

Objectives:

- Solve problems using Newton's Law of Cooling

Cooling to Room Temperature



Notation

$$T^*(t) = T_0^* e^{-kt}$$

- $T^*(t)$ = temperature at time t
- T_0^* = initial temperature
- t = time
- k = rate of decay

$$(T_t - T_m) = (T_0 - T_m)e^{-kt}$$

$$T^*(t) = (T_t - T_m)$$

Example

A cup of coffee is heated to 80°C. It is placed in a room whose temperature is 20°C. After 2 minutes, the temperature has decreased to 60°C. At what time will the coffee have cooled to 40°C?

$$T^*(t) = T_0^* e^{-kt}$$

Time	Temp above r.t.
0	$80 - 20 = 60$
2	$60 - 20 = 40$
?	$40 - 20 = 20$

$$40 = 60e^{-2k} \Rightarrow k \approx 0.20273$$

$$20 = 60e^{-0.20273t} \Rightarrow t \approx 5.42 \text{ min.}$$

Recap

Newton's Law of Cooling

$T^*(t) = T_0^* e^{-kt}$

All temperatures are measured as temperature above room temperature.

Details of Example

$$T^*(t) = T_0^* e^{-kt}$$

$$\Rightarrow 40 = 60e^{-2k}$$

$$\Rightarrow \frac{40}{60} = e^{-2k}$$

$$\Rightarrow \ln\frac{40}{60} = -2k$$

$$\Rightarrow \ln\frac{60}{40} = 2k$$

$$\Rightarrow k = \frac{\ln 3/2}{2}$$

$$\Rightarrow k \approx 0.20273$$

Details of Example

$$T^*(t) = T_0^* e^{-kt}$$

$$\Rightarrow 20 = 60e^{-0.20273t}$$

$$\Rightarrow \frac{20}{60} = e^{-0.20273t}$$

$$\Rightarrow \ln\frac{20}{60} = -0.20273t$$

$$\Rightarrow \ln 3 = 0.20273t$$

$$\Rightarrow t = \frac{\ln 3}{0.20273}$$

$$\Rightarrow t \approx 5.42$$