

## Radioactive Decay



## Preliminaries and Objectives

Preliminaries:

- Standard model for exponential growth and decay

$$A(t) = A_0 e^{rt}$$

- Conversion between logarithmic form and exponential form

Objectives:

- Solve problems involving radioactive decay and half-life

## Notation

$$N(t) = N_0 e^{-\lambda t}$$

- $N(t)$  = amount present at time  $t$
- $N_0$  = initial amount
- $t$  = time in years
- $\lambda$  = rate of decay

## Half-life calculations

The half-life of Rhodium-101 is 3.3 years. If 100 g are present initially, how much will remain after 1 year?

$$N(t) = N_0 e^{-\lambda t}$$

$$\frac{N(t)}{N_0} = \frac{1}{2} = e^{-\lambda(3.3)}$$

$$\Rightarrow \ln \frac{1}{2} = -3.3\lambda$$

$$\Rightarrow -\ln 2 = -3.3\lambda \Rightarrow \ln 2 = 3.3\lambda \Rightarrow \frac{\ln 2}{3.3} = \lambda$$

## Half-life calculations

If  $H$  is the half-life, then

$$\lambda = \frac{\ln 2}{H}$$

## Example 1

The half-life of Rhodium-101 is 3.3 years. If 100 g are present initially, how much will remain after 1 year?

$$\lambda = \frac{\ln 2}{H} = \frac{\ln 2}{3.3} \approx .2100$$

$$N(1) \approx 100e^{-(.2100)(1)} \approx 81.05g$$

## Example 2

After 3 years, 86% of a radioactive element remains. What is its half-life?

$$N(t) = N_0 e^{-\lambda t}$$

$$\frac{N(t)}{N_0} = .86 = e^{-\lambda(3)}$$

$$\ln(.86) = -3\lambda$$

$$\lambda \approx .05027$$

$$\lambda = \frac{\ln 2}{H} \Rightarrow .05027 \approx \frac{\ln 2}{H} \Rightarrow H \approx 13.78 \text{ years}$$

## Example 3

Carbon-14 has a half-life of 5730 years. Living organisms keep replenishing their supply of Carbon-14 from the atmosphere. Once they die, the Carbon-14 begins to decay. A sample from a living organism contains 14.0 g of Carbon-14. A similar sample from a deceased organism contains 12.8 g. How long ago did the organism die?

$$\lambda = \frac{\ln 2}{H} = \frac{\ln 2}{5730} \approx .00012097$$

$$N(t) = N_0 e^{-\lambda t} \Rightarrow 12.8 = 14.0 e^{-.00012097t}$$

$$t \approx 741 \text{ years}$$

## Recap

- $N(t) = N_0 e^{-\lambda t}$

- If  $H$  is the half-life, then

$$\lambda = \frac{\ln 2}{H}$$

which follows from

$$\frac{1}{2} = e^{-\lambda H}$$