Compounding Interest

Preliminaries and Objectives

Preliminaries:
• Standard model for exponential growth and decay
  \[ A(t) = A_0 e^{rt} \]
• Conversion between logarithmic form and exponential form

Objectives:
• Solve problems involving continuously compounded interest.

Notation

\[ A = P e^{rt} \]

- \( A \) = Accrual, the value at time \( t \)
- \( P \) = Principal
- \( t \) = time in years
- \( r \) = interest rate (APR)

Example 1

If $6000 is invested at 3% interest for 7 years, how much will the investment be worth at the end of the investment period?

\[ A = 6000 e^{(0.03)(7)} \approx 7402.06 \]
Example 2

How much need to be invested now so that an investment at 5% interest will be worth $25,000 in three years?

\[ A = Pe^{rt} \]

\[ 25000 = Pe^{0.05(3)} = Pe^{15} \]

\[ P = \frac{25000}{1.1618} \approx 21517.70 \]

Example 3

At what interest would you need to invest $100,000 so that in 25 years, the investment would be worth $500,000?

\[ A = Pe^{rt} \]

\[ 500000 = 100000e^{r(25)} \]

\[ \frac{500000}{100000} = e^{25r} \]

\[ 5 = e^{25r} \]

\[ \ln 5 = 25r \]

\[ r \approx 6.44\% \]

Example 4

How long would you need to invest $3,000 at 4% interest so that at the end of the investment period, it would be worth $5,000?

\[ A = Pe^{rt} \]

\[ 5000 = 3000e^{0.04(t)} \]

\[ \frac{5000}{3000} = e^{0.04t} \]

\[ 5 = 3e^{0.04t} \]

\[ \ln 5 = 0.04t \]

\[ t \approx 12.77 \text{ years} \]

Recap

- \( A = Pe^{rt} \)

- To solve for \( r \) or \( t \), change from exponential form to logarithmic form