Radioactive Decay

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Preliminaries and Objectives

Preliminaries:

- Standard model for exponential growth and decay

$$A(t) = A_0 e^{rt}$$

- Conversion between logarithmic form and exponential form

Objectives:

- Solve problems involving radioactive decay and half-life

Notation

$$N(t) = N_0 e^{-\lambda t}$$

- $N(t)$ = amount present at time t
- N_0 = initial amount
- t = time in years
- λ = rate of decay

Half-life calculations

The half-life of Rhodium-101 is 3.3 years. If 100 g are present initially, how much will remain after 1 year?

$$N(t) = N_0 e^{-\lambda t}$$

$$\frac{N(t)}{N_0} = \frac{1}{2} = e^{-\lambda(3.3)}$$

$$\Rightarrow \ln\frac{1}{2} = -3.3\lambda$$

$$\Rightarrow -\ln 2 = -3.3\lambda \Rightarrow \ln 2 = 3.3\lambda \Rightarrow \frac{\ln 2}{3.3} = \lambda$$

Half-life calculations

If H is the half-life, then

$$\lambda = \frac{\ln 2}{H}$$

Example 1

The half-life of Rhodium-101 is 3.3 years. If 100 g are present initially, how much will remain after 1 year?

$$\lambda = \frac{\ln 2}{H} = \frac{\ln 2}{3.3} \approx .2100$$

$$N(1) \approx 100e^{-(.2100)(1)} \approx 81.05g$$

Example 2

After 3 years, 86% of a radioactive element remains. What is its half-life?

$$N(t) = N_0 e^{-\lambda t}$$

$$\frac{N(t)}{N_0} = .86 = e^{-\lambda(3)}$$

$$\ln(.86) = -3\lambda$$

$$\lambda \approx .05027$$

$$\lambda = \frac{\ln 2}{H} \Rightarrow .05027 \approx \frac{\ln 2}{H} \Rightarrow H \approx 13.78 \text{ years}$$

Example 3

Carbon-14 has a half-life of 5730 years. Living organisms keep replenishing their supply of Carbon-14 from the atmosphere. Once they die, the Carbon-14 begins to decay. A sample from a living organism contains 14.0 g of Carbon-14. A similar sample from a deceased organism contains 12.8 g. How long ago did the organism die?

$$\lambda = \frac{\ln 2}{H} = \frac{\ln 2}{5730} \approx .00012097$$

$$N(t) = N_0 e^{-\lambda t} \Rightarrow 12.8 = 14.0e^{-.00012097t}$$

$$t \approx 741 \text{ years}$$

Recap

- $N(t) = N_0 e^{-\lambda t}$
- If H is the half-life, then
$$\lambda = \frac{\ln 2}{H}$$
which follows from
$$\frac{1}{2} = e^{-\lambda H}$$