Radioactive Decay

Preliminaries and Objectives

Preliminaries:
- Standard model for exponential growth and decay
  \[ A(t) = A_0 e^{rt} \]
- Conversion between logarithmic form and exponential form

Objectives:
- Solve problems involving radioactive decay and half-life

Notation

\[ N(t) = N_0 e^{-\lambda t} \]

- \( N(t) \) = amount present at time \( t \)
- \( N_0 \) = initial amount
- \( t \) = time in years
- \( \lambda \) = rate of decay

Half-life calculations

The half-life of Rhodium-101 is 3.3 years. If 100 g are present initially, how much will remain after 1 year?

\[
\frac{N(t)}{N_0} = \frac{1}{2} = e^{-\lambda(3.3)}
\]

\[
\Rightarrow \ln \frac{1}{2} = -3.3\lambda
\]

\[
\Rightarrow -\ln 2 = -3.3\lambda \Rightarrow \ln 2 = 3.3\lambda \Rightarrow \frac{\ln 2}{3.3} = \lambda
\]
## Half-life calculations

If $H$ is the half-life, then

$$\lambda = \frac{\ln 2}{H}$$

### Example 1

The half-life of Rhodium-101 is 3.3 years. If 100 g are present initially, how much will remain after 1 year?

$$\lambda = \frac{\ln 2}{H} = \frac{\ln 2}{3.3} \approx .2100$$

$$N(1) \approx 100e^{-(.2100)(1)} \approx 81.05g$$

### Example 2

After 3 years, 86% of a radioactive element remains. What is its half-life?

$$N(t) = N_0 e^{-\lambda t}$$

$$\frac{N(t)}{N_0} = .86 = e^{-\lambda(3)}$$

$$\ln(.86) = -3\lambda$$

$$\lambda \approx .05027$$

$$\lambda = \frac{\ln 2}{H} \Rightarrow .05027 \approx \frac{\ln 2}{H} \Rightarrow H \approx 13.78 \text{ years}$$

### Example 3

Carbon-14 has a half-life of 5730 years. Living organisms keep replenishing their supply of Carbon-14 from the atmosphere. Once they die, the Carbon-14 begins to decay. A sample from a living organism contains 14.0 g of Carbon-14. A similar sample from a deceased organism contains 12.8 g. How long ago did the organism die?

$$\lambda = \frac{\ln 2}{H} = \frac{\ln 2}{5730} \approx .00012097$$

$$N(t) = N_0 e^{-\lambda t} \Rightarrow 12.8 = 14.0e^{-.00012097t}$$

$$t \approx 741 \text{ years}$$
Recap

- $N(t) = N_0 e^{-\lambda t}$

- If $H$ is the half-life, then
  \[ \lambda = \frac{\ln 2}{H} \]

  which follows from
  \[ \frac{1}{2} = e^{-\lambda H} \]