Solving 3 x 3 Systems of Linear Equations
Preliminaries and Objectives

Preliminaries

• Solving a 2 x 2 system of linear equations
  • Substitution Method
  • Elimination Method

Objectives

• Find the solution to a system of 3 equations in three variables.
Solving 2 x 2 Systems of Equations - Elimination Method

- Multiply one or both equations by a constant so that one variable will cancel.
- Add equations together to get new equation with one variable.
- Solve for first variable.
- Substitute to find second variable.
Solving 3 x 3 Systems of Equations

- Pick two of the three equations and multiply one or both equations by a constant so that one variable will cancel.
- Add equations together to get new equation with two variables.
- Pick a different pair of equations and multiply one or both equations by a constant so that the same variable will cancel.
- Add equations together to get new equation with the same two variables.
- Solve the new 2 x 2 system
Example 1

(I) \( x + y + z = 0 \)
(II) \( -2x + 2y - 4z = 12 \)
(III) \( 2x - 3y - z = 7 \)

\[ 2 \times (I) \]

(I) \( 2x + 2y + 2z = 0 \)
(II) \( -2x + 2y - 4z = 12 \)

(IV) \( 4y - 2z = 12 \)

(V) \( -y - 5z = 19 \)
Example 1

\[(IV) \quad 4y - 2z = 12\]
\[(V) \quad -y - 5z = 19\]

\[4 \times (V) \quad 4y - 20z = 76\]
\[\begin{array}{c}
-22z = 88 \\
n\end{array}\]
\[z = -4\]
\[(V) \quad -y + 20 = 19\]
\[y = 1\]
\[(I) \quad x + y + z = 0\]
\[(I) \quad x + 1 - 4 = 0\]
\[x = 3\]

\[(3, 1, -4)\]
Geometry of Linear Equations in 3-dimensional space

- Each linear equation is the equation of a plane.
- Two planes intersect in a line (usually).
- A line intersects the third equation (plane) at a point (usually).
- A false statement means two planes were parallel and there are no points of intersection of all three planes.
- A true statement means that there are infinitely many solutions, either because two planes were the same plane or that every pair of planes intersects at the same line.
Example 2

\[
\begin{align*}
(I) & \quad -3x + 2y - 4z = 12 \\
(II) & \quad y + 2z = 5 \\
(III) & \quad 2x + 4y + 5z = 2 \\
\end{align*}
\]

\[
\begin{align*}
2 \times (I) & \quad -6x + 4y - 8z = 24 \\
3 \times (II) & \quad 6x + 12y + 15z = 6 \\
IV & \quad 16y + 7z = 30 \\
-16 \times (II) & \quad -16y - 32z = -80 \\
\end{align*}
\]

\[
\begin{align*}
& \quad -25z = -50 \\
& \quad z = 2 \\
& \quad y = 1 \\
& \quad x = -6 \\
\end{align*}
\]

\((-6, 1, 2)\)
Recap

- Eliminate one variable using one pair of equations.
- Eliminate the **same** variable from a different pair of equations.
- Solve the resulting 2 x 2 system of equations.
- Substitute to find the values of the other variables.