Solving 2 x 2 Systems of Linear Equations
Preliminaries and Objectives

Preliminaries

- Graphs of Lines
- Algebraic Skills
  - Distributive Law
  - Combining Like Terms
  - Solving Linear Equations

Objectives

- Determine if two lines intersect
- Find the intersection of two lines
Intersecting Lines

$$3x - 5y = 4$$

$$x = 4y - 1$$
Substitution Method

\[
\begin{align*}
3x - 5y &= 4 \\
x &= 4y - 1
\end{align*}
\]

\[
3(4y - 1) - 5y = 4
\]

\[
12y - 3 - 5y = 4
\]

\[
7y = 7
\]

\[
y = 1
\]

\[
x = 4(1) - 1 = 3
\]
Examples

\begin{align*}
2x - 3y &= -8 \\
\phantom{2x} - 3y &= -8 \\
8 &= 4y \\
5x - y &= 3 \\
3x + 2y &= 20 \\
\phantom{3x} + 2y &= \phantom{20} \\
x &= -1, \quad y = 2 \\
x &= 2, \quad y = 7
\end{align*}
Elimination Method

\[3x - 4y = -3\]
\[5x + 2y = 21\] multiply by 2

\[
\begin{align*}
3x - 4y &= -3 \\
10x + 4y &= 42
\end{align*}
\]

\[13x = 39\]

\[x = 3, \ y = 3\]
Inconsistent Systems

Solving 2 x 2 Systems of Linear Equations

\[ 2x - y = 7 \]
\[ -4x + 2y = 6 \]
\[ 2x - y = 7 \]
\[ -2x + y = 3 \]

No solutions
Dependent Systems

2x − y = 7
4x − 2y = 14

−4x + 2y = −14
4x − 2y = 14

0 = 0

Infinitely many solutions, the lines are the same line.
### Solutions to a 2x2 System of Linear Equations

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