Geometric Sequences and Series
Preliminaries

- Sequences
- Exponential functions

Objectives

- Define a geometric sequence
- Find the sum of an infinite geometric series
A **geometric** sequence is a sequence of numbers in which the recursion is to multiply by a constant, called the *common ratio*. 
Explicit formula

As the input \( n \) changes by 1, the output \( g_n \) doubles

\[
g_n = 3(2^n)
\]
Example 2

\[ \{g_n\} = 9 \{ -3, 1, -\frac{1}{3}, \frac{1}{9}, \ldots \} \quad r = -\frac{1}{3} \]

Find the \( n \)th term.

Find the 5th term.

\[ g_n = 9(-1)^n \left( \frac{1}{3} \right)^n \]

\[ g_5 = 9(-1)^5 \left( \frac{1}{3} \right)^5 = -\frac{1}{27} \]
Example 3

Find the sum

\[ S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{256} \]

\[-\left(\frac{1}{2}\right)S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{256} + \frac{1}{512}\]

\[ \left(\frac{1}{2}\right)S = \frac{1}{2} - \frac{1}{512} \]

\[ \left(\frac{1}{2}\right)S = \frac{255}{512} \]

\[ S = \frac{255}{256} \]
Find the sum

\[ S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \]

\[- (\frac{1}{2}) S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \]

\[ (\frac{1}{2}) S = \frac{1}{2} \]

\[ S = 1 \]
Example 5

Find the sum

\[ S = \frac{3}{2} - \frac{1}{2} + \frac{1}{6} - \frac{1}{18} + \cdots \]

\[ + \left( \frac{1}{3} \right) S = \frac{1}{2} - \frac{1}{6} + \frac{1}{18} + \cdots \]

\[ \left( \frac{4}{3} \right) S = \frac{3}{2} \]

\[ S = \frac{9}{8} \]
Find the sum

\[ S = \frac{3}{5} + \frac{6}{5} + \frac{12}{5} + \frac{24}{5} + \ldots \]

The terms get larger, so the sum does not exist. The series **diverges**.
Example 7

Find the sum

$$\sum_{k=1}^{\infty} \left( -\frac{9}{2} \right) \left( -\frac{1}{3} \right)^{k}$$

$$S = \frac{3}{2} - \frac{1}{2} + \frac{1}{6} - \frac{1}{18} + \cdots$$
Recap

- Explicit formula for a geometric series $g_n = g_0 \cdot r^n$
- To find the sum of a geometric series, multiply the series by the common ratio, then subtract.