Geometric Sequences and Series

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Preliminaries and Objectives

Preliminaries

- Sequences
- Exponential functions

Objectives

- Define a geometric sequence
- Find the sum of an infinite geometric series

Geometric Sequences

A **geometric** sequence is a sequence of numbers in which the recursion is to multiply by a constant, called the *common ratio*.

Explicit formula

n	0	1	2	3	4
g_n	3	6	12	24	48

As the input n changes by 1, the output g_n doubles

$g_n = 3(2^n)$

Example 2

$\{g_n\} = 9\ \{\ -3, 1, -\frac{1}{3}, \frac{1}{9} \ldots\}$ $\quad r = -\frac{1}{3}$

Find the *n*th term.

Find the 5th term.

$$g_n = 9(-1)^n(\tfrac{1}{3})^n$$

$$g_5 = 9(-1)^5(\tfrac{1}{3})^5 = -\frac{1}{27}$$

Example 3

Find the sum

$$
\begin{aligned}
S &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \ldots + \tfrac{1}{256} \\
-\left(\tfrac{1}{2}\right)S &= \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \ldots + \tfrac{1}{256} + \tfrac{1}{512}
\end{aligned}
$$

$$\left(\tfrac{1}{2}\right)S = \tfrac{1}{2} - \tfrac{1}{512}$$

$$\left(\tfrac{1}{2}\right)S = \tfrac{255}{512}$$

$$S = \tfrac{255}{256}$$

Example 4

Find the sum

$$\begin{aligned} S &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \dots \\ -\left(\tfrac{1}{2}\right)S &= \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \dots \end{aligned}$$

$$\left(\tfrac{1}{2}\right)S = \tfrac{1}{2}$$

$$S = 1$$

Example 5

Find the sum

$$\begin{aligned} S &= \tfrac{3}{2} - \tfrac{1}{2} + \tfrac{1}{6} - \tfrac{1}{18} + \dots \\ +(\tfrac{1}{3})S &= \tfrac{1}{2} - \tfrac{1}{6} + \tfrac{1}{18} + \dots \end{aligned}$$

$$(\tfrac{4}{3})S = \tfrac{3}{2}$$

$$S = \tfrac{9}{8}$$

Example 6

Find the sum

$$S = \frac{3}{5} + \frac{6}{5} + \frac{12}{5} + \frac{24}{5} + \dots$$

The terms get larger, so the sum does not exist. The series **diverges**.

Example 7

Find the sum

$$\sum_{k=1}^{\infty} \left(-\frac{9}{2}\right)\left(-\frac{1}{3}\right)^{k}$$

$$S = \tfrac{3}{2} - \tfrac{1}{2} + \tfrac{1}{6} - \tfrac{1}{18} + \dots$$

Recap

- Explicit formula for a geometric series $g_n = g_0 \cdot r^n$
- To find the sum of a geometric series, multiply the series by the common ratio, then subtract.