Compounding Interest

Preliminaries and Objectives

Preliminaries:

- Standard model for exponential growth and decay

$$A(t) = A_0 e^{rt}$$

- Conversion between logarithmic form and exponential form

Objectives:

- Solve problems involving continuously compounded interest.

Notation

$$A = Pe^{rt}$$

- $A =$ Accrual, the value at time t
- $P =$ Principal
- $t =$ time in years
- $r =$ interest rate (APR)

Example 1

If $6000 is invested at 3% interest for 7 years, how much will the investment be worth at the end of the investment period?

$$A = Pe^{rt}$$

$$A = \$6000e^{(.03)(7)} \approx \$7402.06$$

Example 2

How much need to be invested now so that an investment at 5% interest will be worth $25,000 in three years?

$$A = Pe^{rt}$$

$$\$25000 = Pe^{(.05)(3)} = Pe^{.15}$$

$$P \approx \frac{\$25000}{1.1618} \approx \$21517.70$$

Example 3

At what interest would you need to invest \$100,000 so that in 25 years, the investment would be worth \$500,000?

$$A = Pe^{rt}$$

$$\$500000 = \$100000e^{(r)(25)}$$

$$\frac{\$500000}{100000} = \frac{\$100000e^{(r)(25)}}{100000}$$

$$5 = e^{25r}$$

$$\ln 5 = 25r$$

$$r \approx 6.44\%$$

Example 4

How long would you need to invest \$3,000 at 4% interest so that at the end of the investment period, it would be worth \$5,000?

$$A = Pe^{rt}$$

$$\$5000 = \$3000e^{(.04)(t)}$$

$$\frac{5000}{3000} = e^{.04t}$$

$$\ln \frac{5}{3} = .04t$$

$$t \approx 12.77 \text{ years}$$

Recap

- $A = Pe^{rt}$

- To solve for r or t, change from exponential form to logarithmic form