Solving Systems of Non-linear Equations

Preliminaries and Objectives

- Graph of circles, ellipses, parabolas and hyperbolas.
- Transformation of graphs.
- Solving polynomial equations in one variable.

Objectives
- Find the intersection points of polynomial equations.

Solving Polynomial Equations by Factoring

\[ x^2 - 25x + 144 = 0 \]
\[ (x^2 - 9)(x^2 - 16) = 0 \]
\[ (x - 3)(x + 3)(x - 4)(x + 4) = 0 \]
\[ x = \pm 3 \text{ or } x = \pm 4 \]

Solving Polynomial Equations by Completing the Square

\[ (x - 3)^2 = 7 \]
\[ x - 3 = \pm \sqrt{7} \]
\[ x = 3 \pm \sqrt{7} \]

Solving Polynomial Equations by the Quadratic Formula

If \( ax^2 + bx + c = 0 \), then
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Hyperbola - Version 2

\[ xy = 1 \iff y = \frac{1}{x} \]

Example 1 - Substitution

\[ y = 2x^2 \]
\[ y = 2x + 4 \]
\[ 2x^2 = 2x + 4 \]
\[ 2x^2 - 2x - 4 = 0 \]
\[ x^2 - x - 2 = 0 \]
\[ (x - 2)(x + 1) = 0 \]
\[ x = 2 \text{ or } x = -1 \]
\[ (2, 8) \quad (-1, 2) \]
Example 2 - Substitution

\[ xy = 12 \]
\[ x^2 + y^2 = 25 \]

\[ y = \frac{12}{x} \]
\[ x^2 + \frac{144}{x^2} = 25 \]
\[ x^4 + 144 = 25x^2 \]
\[ x^4 - 25x^2 + 144 = 0 \]
\[(x - 3)(x + 3)(x - 4)(x + 4) = 0 \]
\[(3, 4), (4, 3), (-3, -4), (-4, -3)\]

Example 3 - Elimination

\[ \frac{x^2}{4} - \frac{y^2}{9} = 1 \]
\[ \frac{x^2}{9} - y^2 = 1 \]

\[ 9x^2 + y^2 = 9 \]
\[ 85x^2 = 10 \]
\[ x = \pm \frac{\sqrt{2}}{\sqrt{17}} \]
\[ y^2 = -\frac{9}{17} \]

Example 4 - Substitution

\[ 4x^2 + y^2 = 16 \]
\[ y^2 = x + 2 \]
\[ 4x^2 + x + 2 = 16 \]
\[ 4x^2 + x - 14 = 0 \]
\[ (4x - 7)(x + 2) = 0 \]
\[ x = \frac{7}{4} \text{ or } x = -2 \]
\[ (x, y) = \left( \frac{7}{4}, \sqrt{x} \right) \]
\[ \left( \frac{7}{4}, -\sqrt{x} \right) \]

Example 5 - Substitution

\[ y = \sqrt{x} \]
\[ y = x - 2 \]
\[ \sqrt{x} = x - 2 \]
\[ x = x^2 - 4x + 4 \]
\[ x^2 - 5x + 4 = 0 \]
\[ (x - 4)(x - 1) = 0 \]
\[ x = 4 \text{ or } x = 1 \]
\[ (4, 2), (1, -1) \]

Recap

- Substitution and Elimination techniques may be used
- Reduce the equation to a single variable
- Find all solutions for the first variable
- Substitute to find all ordered pairs
- Check solutions by graphing