Preliminaries and Objectives

Preliminaries
- Pascal’s triangle
- Factorials
- Sigma notation
- Expanding binomials

Objectives
- Expand $(x + y)^n$ for $n = 3, 4, 5, \ldots$

Expanding Binomials

$(x + y)^0 = 1$
$(x + y)^1 = 1x + 1y$
$(x + y)^2 = 1x^2 + 2xy + 1y^2$
$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

Notation

The notation for the coefficient of $x^n \cdot y^k$ in the expansion of $(x + y)^n$ is

\[ \binom{n}{k} \]

It is calculated by the following formula

\[ \binom{n}{k} = \frac{n!}{(n-k)!k!} \]

In other words

\[ (x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k \]

Example 1

\[ \binom{7}{4} = \frac{7!}{4!3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1} = 35 \]

Example 2

\[ (x - 2)^7 = x^7 + 7x^6(-2) + 21x^5(-2)^2 + 35x^4(-2)^3 + 35x^3(-2)^4 + 21x^2(-2)^5 + 7x(-2)^6 + (-2)^7 \]

\[ = x^7 - 14x^6 + 84x^5 - 280x^4 + 560x^3 - 672x^2 + 384x - 128 \]

Example 3

\[ (x + y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7 \]
Recap

- The expansion of \((x + y)^n\) has terms whose exponents add to \(n\)
- The coefficient on \(x^k y^{n-k}\) is \(\binom{n}{k} = \frac{n!}{k!(n-k)!}\)