Inverse Functions

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries

- Functions
- Function Notation

Objectives

- Define the inverse of a function

Example 1

Solve for *x*:

$$9 = \sqrt{3x - 6}$$

Step 1 – square both sides

Step 2 – add 6

Step 3 – divide by 3

Notation

Function   Inverse Function

$$f(x) \qquad\qquad f^{-1}(x)$$

Example 2

If $f(x) = x + 3$, find $f^{-1}(x)$

$$y = x + 3 \qquad\qquad x = y + 3$$

$$x - 3 = y$$

$$f^{-1}(x) = x - 3$$

Example 3

If $g(x) = 2x$, find $g^{-1}(x)$

$$y = 2x \qquad\qquad x = 2y$$

$$\frac{x}{2} = y$$

$$g^{-1}(x) = \frac{x}{2}$$

Example 4

If $h(x) = \sqrt{3x - 6}$, find $h^{-1}(x)$

$$y = \sqrt{3x - 6}$$

$$x = \sqrt{3y - 6}$$

$$x^2 = 3y - 6$$

$$x^2 + 6 = 3y$$

$$\frac{x^2 + 6}{3} = y$$

$$h^{-1}(x) = \frac{x^2 + 6}{3}$$

Recap

Procedure to find $f^{-1}(x)$ given $f(x)$:

- Begin with the equation $y = f(x)$

- Interchange x and y

- Solve for y