Exponential Growth and Geometric Sequences

Preliminaries and Objectives

Preliminaries

- Sequences
- Linear Growth
- Arithmetic Sequences

Objectives

- Find values of a geometric sequence defined explicitly
- Find values of a geometric sequence defined recursively
- Find a recursive formula for a geometric sequence
- Find an explicit formula for a geometric sequence

Recursive Definition

Let $g_1 = 5$ and $g_{n+1} = (10)(g_n)$

$$\underline{5} \quad \underline{50} \quad \underline{500} \quad \underline{5000} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

Explicit Definition

Let $g_n = (16)\left(\frac{1}{2}\right)^n$

$$\underline{16} \quad \underline{8} \quad \underline{4} \quad \underline{2} \quad \underline{1} \quad \underline{\frac{1}{2}} \quad \underline{\frac{1}{4}} \quad \underline{\frac{1}{8}}$$

Finding the formulas

Given the geometric sequence

$$\underline{3} \quad \underline{12} \quad \underline{48} \quad \underline{192} \quad \underline{768} \quad \underline{3072}$$

find a recursive formula and an explicit formula.

Recursive Formula:

$g_0 = 3$, $g_{n+1} = (4)(g_n)$

Explicit Formula:

$g_n = 3(4^n)$

Example 1

If a geometric sequence contains the terms $g_3 = 2$ and $g_7 = 162$, find a recursive formula and an explicit formula for g_n.

$$\underline{\tfrac{2}{27}} \quad \underline{\tfrac{2}{9}} \quad \underline{\tfrac{2}{3}} \quad \underline{2} \quad \underline{6} \quad \underline{18} \quad \underline{54} \quad \underline{162} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \cdots$$
$$g_1 \qquad\qquad g_3 \qquad\qquad\qquad\qquad g_7$$

$r =$ 4th root of $\dfrac{162}{2} =$ 4th root of $81 = 3$

Recursive definition: $g_1 = \frac{2}{9}$; $g_{n+1} = (3)(g_n)$

Explicit Definition: $g_n = \frac{2}{27}(3^n)$

Example 2

You put a $500 purchase on your credit card. Each month, interest is compounded until at the end of 9 months, you owe $571.70. What is the interest rate?

$g_0 = 500, g_9 = 571.70$

$$g_9 = g_0(r^9) \Rightarrow r^9 = \frac{571.70}{500} = 1.1434 \Rightarrow r = 1.015$$

The interest rate is 1.5% per month or 18% annually.

Recap

- Recursive definition: State the value of g_0 and the recursion $g_{n+1} = (r)(g_n)$
- Explicit definition: $g_n = (g_0)r^n$