Preliminaries and Objectives

Preliminaries
- Graph of circles, ellipses, parabolas and hyperbolas.
- Transformation of Graphs
- Solving polynomial equations in one variable.

Objectives
- Find the intersection points of polynomial equations.

Solving Systems of Non-linear Equations

Solving Polynomial Equations by Factoring

\[ 4x^2 + x - 14 = 0 \]
\[ (4x - 7)(x + 2) = 0 \]
\[ 4x - 7 = 0 \text{ or } x + 2 = 0 \]
\[ x = \frac{7}{4} \text{ or } x = -2 \]

\[ x^4 - 25x^2 + 144 = 0 \]
\[ (x^2 - 9)(x^2 - 16) = 0 \]
\[ (x - 3)(x + 3)(x - 4)(x + 4) = 0 \]
\[ x = \pm 3 \text{ or } x = \pm 4 \]
Solving Polynomial Equations by Completing the Square

\[(x - 3)^2 = 7\]

\[x - 3 = \pm \sqrt{7}\]

\[x = 3 \pm \sqrt{7}\]

Solving Polynomial Equations by the Quadratic Formula

If \(ax^2 + bx + c = 0\), then

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

Hyperbola - Version 2

\[xy = 1 \iff y = \frac{1}{x}\]

Example 1 - Substitution

\[y = 2x^2\]

\[y = 2x + 4\]
Example 1 - Substitution

\[
y = 2x^2 \\
y = 2x + 4 \\
2x^2 = 2x + 4 \\
2x^2 - 2x - 4 = 0 \\
x^2 - x - 2 = 0 \\
(x - 2)(x + 1) = 0 \\
x = 2 \text{ or } x = -1 \\
(2, 8) \quad (-1, 2)
\]

Example 2 - Substitution

\[
xy = 12 \\
x^2 + y^2 = 25
\]

\[
y = \frac{12}{x} \\
x^2 + \left(\frac{12}{x}\right)^2 = 25 \\
x^2 + \frac{144}{x^2} = 25 \\
x^4 + 144 = 25x^2 \\
x^4 - 25x^2 + 144 = 0 \\
(x - 3)(x + 3)(x - 4)(x + 4) = 0 \\
(3, 4), (4, 3), (-3, -4), (-4, -3)
\]

Example 3 - Elimination

\[
\frac{x^2}{4} + \frac{y^2}{9} = 1 \\
\frac{x^2}{9} - y^2 = 1
\]
Example 3 - Elimination

\[
\begin{align*}
x^2 + \frac{y^2}{9} &= 1 \\
x^2 - y^2 &= 1 \\
\frac{9x^2}{4} + y^2 &= 9 \\
\frac{85x^2}{36} &= 10 \\
x &= \pm \sqrt{\frac{72}{17}} \\
y^2 &= -\frac{9}{17}
\end{align*}
\]

Example 4 - Substitution

\[
\begin{align*}
4x^2 + y^2 &= 16 \\
y^2 &= x + 2 \\
4x^2 + x + 2 &= 16 \\
4x^2 + x - 14 &= 0 \\
(4x - 7)(x + 2) &= 0 \\
x &= \frac{7}{4} \text{ or } x = -2 \\
(-2, 0), \left(\frac{7}{4}, \frac{\sqrt{15}}{2}\right), \left(\frac{7}{4}, -\frac{\sqrt{15}}{2}\right)
\end{align*}
\]

Example 5 - Substitution

\[
\begin{align*}
y &= \sqrt{x} \\
y &= x - 2 \\
\sqrt{x} &= x - 2 \\
x &= x^2 - 4x + 4 \\
x^2 - 5x + 4 &= 0 \\
(x - 4)(x - 1) &= 0 \\
x &= 4 \text{ or } x = 1 \\
(4, 2), (1, -1)
\end{align*}
\]

Recap

- Substitution and Elimination techniques may be used
- Reduce the equation to a single variable
- Find all solutions for the first variable
- Substitute to find all ordered pairs
- Check solutions by graphing