

Examples Using the Properties of Logarithms



Preliminaries and Objectives

Preliminaries

- Laws of Logarithms
- Laws of Exponents

Objectives

- Simplify expressions using logarithms
- Solve equations involving logarithms

Laws of Logarithms

$$b^m = x$$

$$b^n = y$$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b b^m = m$$

$$b^{\log_b x} = x$$

$$\log_b x = m$$

$$\log_b y = n$$

$$\log_b \frac{1}{x} = -\log_b x$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^n = n \cdot \log_b x$$

Example 4

Write $2 \log_{10} 3$ as a single logarithm

$$2 \log_{10} 3 = \log_{10} 3^2 = \log_{10} 9$$

Example 5

Write as a single logarithm

$$\log_{10} 7 + \log_{10} 4 = \log_{10}(7)(4) = \log_{10} 28$$

Example 6

$$\begin{aligned} & 3 \log_{10} 6 - 2 \log_{10} 5 \\ &= \log_{10} 6^3 - \log_{10} 5^2 \\ &= \log_{10} 216 - \log_{10} 25 \\ &= \log_{10} \frac{216}{25} \\ &= \log_{10} 8.64 \end{aligned}$$

Example 7

$$3^{\log_3 10 - \log_3 7}$$

$$= 3^{\log_3 \frac{10}{7}}$$

$$= \frac{10}{7}$$

Example 8

Solve for x

$$\log_4 x = 2$$

Ans: $4^2 = x$
 $x = 16$

Example 9

Solve for x

$$\log_{10}(3x + 1) = 1$$

$$\begin{aligned}\text{Ans: } 10^1 &= 3x + 1 \\ x &= 3\end{aligned}$$

Example 10

Solve for x

$$\log_4(x + 2) = \log_4 8$$

Ans: $x + 2 = 8$
 $x = 6$

Example 11

Solve for x

$$\log_{10} 2x - \log_{10}(x - 3) = 1$$

$$\text{Ans: } \log_{10} \frac{2x}{x - 3} = 1$$

$$10^1 = \frac{2x}{x - 3}$$

$$10(x - 3) = 2x$$

$$10x - 30 = 2x$$

$$8x = 30$$

$$x = \frac{30}{8}$$