1. Radioactive Decay

2. You should be familiar with the standard model for exponential growth and decay, the amount of some quantity present at time t is the initial amount times e to a power, where r is the growth rate and t is time. You should also be familiar with solving exponential equations, including the conversion from exponential form to logarithmic form. In this lesson, we will solve problems involving radioactive decay and half-life.

3. Here is an example: The half-life of Rhodium-101 is 3.3 years. If 100 grams are present initially, how much will remain after one year?

4. We use a slightly different notation for radioactive decay, though the basic quantities are the same. $N(t)$ represents the amount present at time t. N_0 is the initial amount. λ is the rate of decay and t is time. Note the negative sign in the exponent. We expect a negative exponent since the amount of material is decreasing. The negative sign ensures that the decay rate is positive.

5. (a) The first thing we need to deal with is the concept of half-life.
   (b) The half-life is the amount of time it takes until half of the radioactive material remains.
   (c) Dividing both sides by the initial amount, the left hand side becomes the amount remaining divided by the initial amount. We want this quantity to be one-half.
   (d) Note that it doesn't matter what number of grams we started with, all that is important for half-life is the ratio of what remains to what we started with. The half-life is the time until half remains. Substituting 3.3 for t, we can now find the decay rate λ.
   (e) To solve, take the log of both sides
   (f) Recall that logs are exponents, and that negative exponents are used for reciprocals, so we can change the sign on the log by taking the reciprocal of 1/2.
   (g) then clear the negative signs
   (h) to find λ

6. This process generalizes: To find λ, divide the log of 2 by the half-life.

7. (a) Returning to the example, we wish to find the amount that remains after 1 year, if we know the half-life
   (b) We begin by finding the decay rate λ
   (c) We then have a straightforward calculation, plugging in the initial amount, the decay rate, and the time.

8. (a) Here is a second example: We know the fraction that remained after a given time, and wish to find the half-life.

   (b) Note that we were not given either the initial amount or the amount that remained, all we were given was the ratio of the two, which is all we need to know. Plugging in $t = 3$, we can find the rate of decay λ.

   (c) We take the log of both sides

   (d) and solve for λ

   (e) Once we know λ, we can use the half-life formula to find the half-life.

9. (a) Here is a third example. Living things replenish their carbon supply from the atmosphere. Once the organism dies, the carbon-14 begins to decay. If we can compare similar samples, we can estimate how long ago the organism died.

   (b) Once again, we use the half-life to find λ, the rate of decay. We then plug in the values for the amount remaining, the initial amount and λ.

   (c) Divide both sides by 14.0, take a log on both sides and solve for t

10. To recap: We use the standard exponential growth model, except that the material is decaying, so the exponent is negative. λ is the decay rate, t is time, $N(t)$ is the amount present at time t and $N(0)$ is the initial amount.

    λ is related to half-life by the formula λ equals the log of 2 divided by the half-life, which follows from the definition of half-life.