

1. Properties of Logarithms
2. You should be familiar with the Laws of Exponents, and with the definition of the logarithm function. In this lesson, we will find Laws of Logarithms that correspond to Laws of Exponents.
3. (a) Recall the definition of the logarithmic function with base  $b$ . If  $b^m = x$ , then  $m$  is the exponent you put on  $b$  to get  $x$ .  
(b) What is  $\log_b$  of 1? For any base  $b$ ,  $b^0 = 1$ , therefore 0 is the exponent you put on  $b$  to get 1.
4. What is  $\log_b$  of  $b$ ? For any base  $b$ ,  $b^1 = b$ , therefore 1 is the exponent you put on  $b$  to get  $b$ .
5. Before we go further, it's time for a riddle: What is the color of the blue balloon? It's not a difficult riddle, the description of the object gives the information necessary to answer the question. The color of the blue balloon is the color blue.
6. There are two rules which combine exponents and logarithms. What is  $\log_b b^m$ ? We can use our definitions to solve this,  $b^m = x$  and  $\log_b x = m$ , but already knew this from the description. The exponent you put on  $b$  to get  $b$  with an exponent of  $m$  is  $m$ .
7. This one is similar.  $b$  raised to the the power  $\log_b x$  can be found using our definitions. We can also get the answer directly,  $b$  is raised to the power, which is the power to which you raise  $b$  to get  $x$ .
8. What is  $\log_b$  of  $1/x$ ? Using the definition,  $x = b^m$  so  $1/x = 1/b^m$  and we know that reciprocals are represented by negative exponents. Therefore, the exponent you put on  $b$  to get  $1/x$  is  $-m$ , which is the opposite of the exponent you put on  $b$  to get  $x$ .
9. We will now define a second exponential and logarithmic equation and ask what happens when we multiply. What exponent do we put on  $b$  to get  $xy$ ? When multiplying numbers, we add exponents. So  $m + n$  is the exponent we put on  $b$  to get  $xy$ .
10. What exponent do we put on  $b$  to get  $x/y$ ? When dividing numbers, we subtract exponents. So  $m - n$  is the exponent we put on  $b$  to get  $x/y$ .
11. Finally, how do we handle exponents within logarithms. Using our definitions,  $x^n = (b^m)^n = b^{mn} = b^{n \cdot m}$ , so  $n$  times  $m$  is the exponent we put on  $b$  to get  $x^n$ .
12. To recap: Recall that logs are exponents. Zero is the exponent you put on anything to get 1. An exponent of 1 leaves the base the same. Exponents and logarithms are inverses, so they undo each other. Negative logs are negative exponents, which correspond to reciprocals. Multiplying numbers corresponds to adding exponents, and therefore to adding logs. Dividing numbers corresponds to subtracting exponents, and therefore to subtracting logs. When raising a power to a power, the exponents are multiplied.