1. Newton's Law of Cooling

2. You should be familiar with the standard model for exponential growth and decay, the amount of some quantity present at time t is the initial amount times e to a power, where r is the growth rate and t is time. You should also be familiar with solving exponential equations, including the conversion from exponential form to logarithmic form. In this lesson, we will solve problems using Newton's Law of Cooling.

3. (a) Newton's Law of Cooling states that the temperature of a heated object, like a cup of coffee, exponentially decays to the temperature of its surroundings, typically called room temperature.
   (b) Suppose room temperature is $20°C$ and initially, the coffee is $80°C$,
   (c) as time passes, the excess heat in the coffee dissipates into the surrounding air.

4. (a) We use the standard exponential decay model, where $T^*(t)$ is the temperature at time t, T_0^* is the initial temperature, t is time, and k is the rate at which the temperature decreases. Since the temperature is decreasing, we use a negative sign in the exponent so that k is a positive rate. But how do we measure temperature? Does it matter if we use Fahrenheit or Celsius or Kelvin? Doesn't the exponential decay model assume that the quantity in question, in this case, the temperature, decay to zero? Certainly we don't mean that the temperature of our coffee is decaying to absolute zero.

   The temperature that we are measuring is the temperature difference between our object and the temperature of the surrounding medium.

   (b) The equation is often written as

$$(T_t - T_m) = (T_0 - T_m)e^{-kt}$$

   T_t is the temperature at time t, T_m is the temperature of the medium in which the hot object is placed, otherwise known as room temperature, T_0 is the initial temperature of the object.

   Each of the temperatures above is the temperature above room temperature. To calculate the temperature above room temperature at time t, we need to know the initial difference between our object and room temperature. To keep things similar to other exponential models, we will use T^* to denote temperature above room temperature.

5. (a) We can solve problems using the exponential model. A cup of coffee is heated to $80°C$. It is placed in a room whose temperature is $20°C$. After 2 minutes, the temperature has decreased to $60°C$. At what time will the coffee have cooled to $40°C$?

   (b) First, recall that the temperature we need to be talking about is the temperature above room temperature, so we should subtract 20 degrees from each measurement.

   (c) We can now use our exponential model to find k using the information when $t = 2$. We use the initial temperature, the time, and the temperature at time $t = 2$ to find that k is approximately 0.20273.

   (d) Once we know the value of k, we can use it, along with the initial temperature and the final temperature to find that the time at which the coffee reached 20 degrees above room temperature is approximately $t = 5.42$, or about 5 minutes and 25 seconds.

   Details of these two calculations are given and the end of the video.

6. To recap: Newton's Law of Cooling states that the difference in temperature between an object and the temperature of its surroundings decays exponentially. We can use the standard exponential decay model. Recall that all temperatures are the difference between the object's temperature and room temperature.