1. The Logarithm Function

2. You should be familiar with exponential functions. You should also be familiar with inverse functions and how the graph of the inverse function relates to the graph of the original function. In this lesson, we will define the inverse to the exponential function, called the logarithm function. We will then rewrite exponential equations using the logarithmic notation and look at properties of the graph of the logarithm function.

3. Let's begin with the exponential function with base 10, that is, the function $y = 10^x$. Anything to the zero power is 1, so when the exponent is 0, the y-value is 1. When the exponent is 1, the y-value is 10. Exponential growth is rapid, when the exponent is 12, the y-value is one trillion. Negative exponents are reciprocals, so when the exponent is negative, the y-value is a fraction less than 1, but always positive.

4. The graph of this exponential function increases from left to right, passes through the point $(0, 1)$, is asymptotic to the x-axis on the negative side (the left), bends upward, has all real numbers for its domain and all positive numbers for its range.

5. We now wish to find an inverse to the exponential function, that is, we wish to find a function which has as it's input various powers of 10, and has as its output, the exponent. This inverse to the exponential function is called the logarithm function. In this case, since the base of the exponential function was 10, we call this the base 10 logarithm.

6. (a) Recall that an inverse function reverses the roles of x and y,
   (b) thus the inverse of the exponential function has y as the exponent.
   (c) The notation for logarithms looks like this. We will often rewrite logarithmic equations as exponential equations. Remember that y, the log, is the exponent.

7. What is the base 10 log of 1000? Let's write this in exponential form. The log is the exponent, so y is the exponent. The base is 10. That leaves only one place to put the number 1000 in the exponential equation. y is the exponent you put on 10 to get 1000. That exponent is 3.

8. Here again is the graph of the exponential function $y = 10^x$.

9. We get the graph of the logarithmic function by reflecting this across the line $y = x$. Each of the properties we had for the exponential function now reverses the roles of x and y. The exponential function went through the point $(0, 1)$, so the log function goes through the point $(1, 0)$. The exponential function approached the negative half of the x-axis from above, that is, from the positive side. The log graph approached the negative half of the y-axis from the positive side, that is, from the right. The domain of the exponential function is the range of the log function, and vice versa. The exponential graph bends sharply upward (the positive y-direction), so the log graph bends sharply in the positive x-direction, that is, to the right.

10. (a) Let's practice some logarithms with other bases, rewriting them in exponential form. Recall that the log is the exponent, so in the first equation, 3 is the exponent. The base is 2, so we write $2^3 = 8$.

    (b) You may wish to pause the video to rewrite the second equation in exponential form.

    (c) The log is the exponent, so x is the exponent you put on b, to get 4.

11. (a) Here are some further problems for you to try. Remember to rewrite the equation in exponential form. In each case, x is equal to a logarithm, and since logs are exponents, x will be the exponent. You may wish to pause the video now to work out these exercises.

    (b)