1. Inverse Functions

2. You should be familiar with functions and function notation. In this lesson, we define the inverse of a function.

3. Often we wish to undo the effect of a mathematical function in order to solve an equation. This is the concept behind the inverse of a function.

4. Symbolically, given a function $f(x)$, we denote the inverse using a superscript -1. The symbol on the right is read 'f-inverse'.

5. (Animation) If a certain function takes x as its input and produces y as its output, the inverse of that function will convert the input y back to the output x.

   We are already familiar with several inverses. Subtraction undoes addition. Division undoes multiplication. For positive numbers, the square root function and the squaring function are inverses. The squaring function takes an input, such as 5 and squares it to get the output 25. The square root function converts 25 back to 5.

   To recap, given a function, an input and corresponding output, the inverse function interchanges the input and output. The input of the function is the output of the inverse and the output of the function is the input of the inverse.

6. (a) To find the inverse of a function symbolically, we reverse the roles of x and y. Suppose $f(x)$ is the function that adds 3.
   (b) We associate the function f with the equation $y = x + 3$.
   (c) We already know that the inverse of this function should subtract 3, but let's find it symbolically. We interchange x and y
   (d) and solve for y.
   (e) The inverse function f^{-1} subtracts 3.

7. (a) Here is an example of undoing multiplication.
   (b) We associate the function g with the equation $y = 2x$
   (c) Interchange x and y,
   (d) and solve for y.
   (e) The inverse function g^{-1} takes the input x and divides by 2.

8. (a) Here is a more involved example. We take the input x, multiply by 3, subtract 6 and take the square root.
   (b) To find the inverse, begin with the equation $y = \sqrt{3x - 6}$
   (c) interchange the roles of x and y and undo the operations to solve for x
   (d) first squaring
   (e) then adding 6
   (f) and finally dividing by 3.
   (g) the inverse function $h^{-1}(x) = \dfrac{x^2 + 6}{3}$
9. To recap: Given $f(x)$, the procedure to find $f^{-1}(x)$ is as follows: begin with an equation where $y =$ the expression involving x. Then reverse the roles of x and y. Finally, solve for y.