1. Geometric Sequences and Series

2. You should be familiar with sequences and exponential functions.

   In this lesson, we will define a geometric sequence and find the sum of an infinite geometric series.

3. A geometric sequence is a sequence of numbers in which the recursion is to multiply by a constant. This is also called exponential growth, the numbers grow at a constant percent each step. Much of the discussion of this topic is similar to equations of exponential functions.

4. (a) For example, the sequence 2,4,8,16 and so on is a geometric sequence. The first term is 2, and each subsequent term is twice the previous term. That is, the common ratio, r is 2.

   (b) We can find an explicit formula for a geometric sequence similar to an exponential function.

   (c) In this case, we can divide by 2 to get a starting value of 3. The 3 is not a member of the sequence. Think of 3 as the starting point, like the y-intercept of a function, and the members of the sequence happen as we step forward.

   (d) The sequence is geometric, because as we step forward to the next term, the output is doubled.

   (e) The formula for the output g_n is now the starting value of 3, multiplied by 2^n.

5. (a) Here is an alternating example. The common ratio is $-\frac{1}{3}$.

   (b) To get back to the starting point, divide -3 by $-\frac{1}{3}$, to get 9.

   (c) To find the nth term, we multiply the starting value of 9 by the common ratio of $-\frac{1}{3}$

   (d) We can separate the common ratio into two components. The $(-1)^n$ makes the sequence alternate, and the $\frac{1}{3}$ makes the sequence shrink from the starting value of 9.

   (e) We can find the fifth term by plugging 5 into the formula. We also could have multiplied the fourth term by $-\frac{1}{3}$.

6. (a) We may also want to find the sum of a geometric series. The technique used here is to multiply the series by the common ratio. In this case, we multiply $\frac{1}{2}$.

   (b) Notice that every term in the series shifted one spot to the right. We can now subtract the two series.

   (c) Everything cancels except for the first term and the last term.

   (d) We can then solve for S

   (e) .

7. (a) We can even find the sum of an infinite series. In the previous example, if instead of stopping at the eighth term, we continued to the one-thousandth term, the number subtracted would have been extremely small, and the final answer extremely close to one. When we go infinitely far, and the terms are shrinking to zero, all we have left is the first term.

(b) We use the same technique, multiplying the series by the common ratio of $\frac{1}{2}$

(c) Then subtracting. Notice that every term after the first will cancel.

(d) We then solve for S

8. (a) Here is an alternating sum.

   (b) Again we multiply by the common ratio of $\frac{1}{3}$. In this case the terms will cancel if we add.

   (c) Again, every term except the first cancels

   (d) and we can solve for S

9. Recall that we rely on the fact that the terms shrink to zero as we go to the right. If the common ratio is bigger than 1, the terms will grow. The sum will also tend to infinity. In this case, we say the sequence diverges.

10. (a) We can write a geometric series in sigma notation. To find the sum, expand the series

    (b) then use the standard technique.

11. To recap: A geometric series is similar to an exponential function. The explicit formula will be the starting value multiplied by the common ratio to the nth power. To find the sum of a geometric series, multiple the entire sum by the common ratio, and subtract.