1. Exponential Growth and Geometric Sequences

2. You should be familiar with sequences of numbers and it may be helpful to understand the commonly used notation. It is also helpful to understand linear growth and arithmetic sequences which are based on an additive pattern. This lesson is similar except that we will use multiplication at each step, instead of addition.

   In this lesson, we will define geometric sequences, both explicitly and recursively, and find both explicit and recursive formulas from the sequence of numbers.

   (Animation)

   Exponential growth occurs when a quantity increases in size through multiplication. The simplest example is something that doubles each step. Suppose you have a soybean field infected with a certain type of insect pest, and each day, the infected parcel of land doubles in size.

   The infestation has doubled after one day.

   Then doubles again

   and again

   and again

   This process continues

   until the entire field is contaminated.

   (end Animation)

3. The notation used is similar to the notation for arithmetic sequences, except that this time, in the recursive part of the definition, we are multiplying, instead of adding. In this sequence, we see that we can get from one term to the next by multiplying by 10. Writing the sequence from the definition requires that we know an initial term, in this case 5, and then getting the next term, by taking the previous term and multiplying by 10.

4. (a) The explicit formula will require us count the repeated multiplication with an exponent. To get the first term, we plug the value 1 in for n to get 16 times one-half, which is 8.

   (b) If you prefer, you could start with the initial term g_0

5. (a) Let's go in the other direction. Given the sequence, find the formulas, both recursive and explicit. The recursive form is easier to find. Recall, you must give two pieces of information in a recursive definition, you must specify the initial term, in this case, $g_1 = 12$. You must also specify how we get from one term to the next. For a geometric sequence, the recursion will be multiplying, so we need to find the common ratio. We can take any two consecutive terms and divide. In this case, we see that each term is 4 times the previous term, so the recursive step will be to multiply by 4.

   (b) Specifically, $g_1 = 12$ and the next term $g_{n+1} = 4g_n$.

(c) to find the explicit formula, it is again helpful to find the value prior to the first term. If we divide once more by 4, we get $g_0 = 3$. This allows us to have already performed one multiplication for the first term. The explicit formula will begin with 3, then count the number of times we multiply by 4 in the exponent.

(d) The explicit formula is g_n equals the initial term of 3 times 4^n.

6. (a) Let's find formulas for this geometric sequence. $g_3 = 2$ and $g_7 = 162$. 162 divided by 2 is 81, so we accomplish a multiplication by 81 in four steps. In order to find the common ratio, we need to find a number we can multiply by itself four times to get 81. In other words, we need to find the 4th root of 81, which is 3.

   (b) The common symbol for the common ratio is the letter r. Now that we have the common ratio, we can start filling in other terms, beginning at 2, and multiplying by 3 each step to the right.

   (c) We can divide by 3 as we go to the left. $g_1 = 2/9$. For the explicit formula, we may wish to find $g_0 = 2/27$. The recursive definition gives the initial term $g_1 = 2/9$ and the recursion where the next term is the previous term times the common ratio of 3.

   (d) The explicit formula starts at 2/27, and then counts the number of steps, which is the number of times we multiply by 3, which is the exponent on 3.

7. (a) Here is another example of a geometric sequence. The application of compounding interest. You put a \$500 purchase on your credit card, and at the end of nine months, you owe \$571.70. What is the interest rate?

   (b) This can be thought of as a geometric sequence with the initial term $g_0 = \$500$, and the ninth term \$571.70, one term for each month. We get to each month by multiplying by the common ratio.

   (c) After multiplying 9 times, your loan has increased by a factor of 1.1434. Taking the 9th root of 1.1434, we find the common ratio of 1.015

   (d) The 1 in the multiplying factor tells us that you still owe the initial \$500. The number after the decimal implies that the interest rate is 1.5% per month, or 18% annually.

8. To recap: Geometric Sequences rely on repeated multiplication. The recursive definition specifies an initial value and a recursion, which determines the next term by multiplying the previous term by the common ratio. The explicit definition multiplies g_0 by the common ratio once for each step, so the number of steps is the exponent.