1. Examples Using the Properties of Logarithms

2. You should be familiar with the Laws of Exponents and the Laws of Logarithms. In this lesson, we will simplify expressions and solve equations using the Laws of Logarithms.

3. Recall the Laws of Logarithms. Zero is the exponent you put on \( b \) to get 1. One is the exponent you put on \( b \) to get \( b \). The exponent you put on \( b \) to get \( b^m \) is \( m \). If the exponent you put on \( b \) to get \( x \) is put on \( b \), you get \( x \). A negative exponent takes the reciprocal of the number. Adding exponents corresponds to multiplying numbers. Subtracting exponents corresponds to dividing numbers. Raising a power to a power corresponds to multiplying exponents.

4. What is the exponent you put on 5 to get 25? We should be able to figure this one out using exponential notation.

5. What is the exponent you put on 5 to get 10? This is harder. It should be clear that the answer is bigger than 1, since \( 5^1 = 5 \), and that the answer is smaller than 2, since \( 5^2 = 25 \).

6. What is the exponent you put on 5 to get \( \sqrt{5} \)? Recall that fractional exponents are roots, so this exponent should be \( 1/2 \).

7. Now let’s use the laws of logarithms. We can rewrite the 2 in front as an exponent on the number 3 to simplify.

8. Here we are adding logs. Logs are exponents, so adding logs is adding exponents. We add exponents when we multiply numbers, so we multiply 7 by 4.

9. Here we combine the rules, first rewriting the exponents. Subtracting logs is subtracting exponents, which is dividing numbers.

10. Here, subtracting logs is subtracting exponents, which is dividing numbers. The exponential and log functions are inverses, and therefore cancel out.

11. (a) We can rewrite this problem in exponential form.
(b) We see that the answer is 16.

12. (a) Likewise, we rewrite in exponential form.
(b) This gives us the equation of a line, that we can solve.

13. (a) Here, the logs have the same base, so the numbers must be the same.
(b) We set the numbers equal to each other and solve.

14. (a) Subtracting logs is subtracting exponents which is dividing numbers.
(b) After using the law of logarithms to simplify, we rewrite in exponential form, cross-multiply, distribute, and solve.