1. Compounding Interest

2. You should be familiar with compounding interest in discrete time steps, like compounding interest annually or monthly, though we will not use that model in this lesson. You should be familiar with the standard model for exponential growth and decay, the amount of some quantity present at time t is the initial amount times e to a power, where r is the growth rate and t is time. This model applies when interest is compounded continuously. You should also be familiar with solving exponential equations, including the conversion from exponential form to logarithmic form. In this lesson, we will solve problems involving continuously compounded interest.

3. The notation used in finance is

   $$A = Pe^{rt}$$

   A stands for the accrual, the amount that an investment is worth in the future.

   P is the principal, the amount of money that is invested initially.

   t is time, measured in years

   r is the annual interest rate, otherwise known as the Annual Percentage Rate, or APR. Note that the APR is not the percent interest you would receive if interest were compounded at this rate once at the end of the year. The percent gain of an investment is slightly higher, and is called the Annual Percentage Yield (APY).

4. (a) Here is an example: If \$6000 is invested at 3% interest for 7 years, how much will the investment be worth at the end of the investment period?

   (b) This is a straightforward calculation. The investment will be worth \$7,402.06 Note that banks will always round off in their favor.

5. (a) Example 2: How much do we need to invest now so that an investment at 5% interest will be worth \$25,000 in three years?

   (b) We use the same formula.

   (c) To find the answer, divide 25000 by e to the .15 power to get \$21517.70

6. (a) Example 3: At what interest would you need to invest \$100,000 so that in 25 years, the investment would be worth \$500,000?

   (b) Here, we plug in the values for A, P and t.

   (c) Divide both sides by $100,000$, the key fact is that we want our investment to grow until it is five times the principal. We now have an exponential equation, which we need to rewrite in logarithmic form by taking a log on both sides.

   (d) $25r$ is the exponent you put on e to get 5.

   (e) Using a calculator the find the natural log of 5, then dividing by 25 finds the interest rate of 6.44%

7. (a) How long would you need to invest \$3,000 at 4% interest so that at the end of the investment period, it would be worth \$5,000?
   (b) Plug in the values for A, P and r
   (c) Divide by 3000
   (d) Rewrite in exponential form, $.04t$ is the exponent you put on e to get $5/3$.
   (e) evaluating the log and dividing by .04 gives $t = 12.774$ years.

8. To recap: Using the formula $A = Pe^{rt}$, to find either A or P, evaluate the exponential function and solve. To find r or t, we will need to take the natural log of both sides.