1. Binomial Theorem

2. You should be familiar with Pascal's triangle, factorials, Sigma notation and expanding binomials by FOILing.

   In this lesson, we will expand higher powers of binomials

3. We wish to look at the pattern when we expand $x + y$ to a power. Anything to the zero power is 1. $(x + y)^1$ is itself $(x + y)$. $(x + y)^2$ has a middle term of $2xy$. When we expand $(x + y)^3$, we get eight terms.

4. (a) When we look just at the coefficients from left to right, we get the familiar pattern from Pascal's triangle. Each row begins and ends with one. The numbers in between are found by adding the two numbers above. For example, 4 plus 6 gives ten. You may wish to stop the video here to write out the next row of Pascal's triangle.
   (b) Here is the seventh row.

5. (a) Let's show why the recursion in Pascal's triangle works for expanding binomials. Starting with the expansion for $(x + y)^3$, let's find $(x + y)^4$.
   (b) We multiply $(x + y)^3$ by $(x + y)$. The first term will be x^4.
   (c) The next two terms will involve x^3y.
   (d) The total will come from adding two coefficients, the coefficients corresponding to the previous row of Pascal's triangle. We also get two terms involving x^2y^2.
   (e) Two terms involving xy^3,
   (f) and finally y^4.
   (g) When we add like terms, we are adding consecutive terms of the previous expansion, exactly the same way entries are added in Pascal's triangle.
   (h)

6. (a) The notation for the coefficients in Pascal's triangle looks like a fraction without the fraction bar. It is read 'n choose k' which comes from another use of Pascal's triangle for counting the number of combinations of objects, a useful building block for probability.
   (b) n choose k is calculated from the following factorial formula.
   (c) The expansion of $(x + y)^n$ is the sum of terms with a coefficient and powers of x and y.

7. Here is an example of calculating 7 choose 4.

8. Also recall that 7 choose 4 is found in Pascal's triangle. Find the row that begins 1-7, then count the coefficients from left to right, starting with 0, 1, 2, 3, 4, the corresponding number is 35. This will be the coefficient on x^4y^3.

9. Here is the full expansion of $(x + y)^7$ using the 7th row of Pascal's triangle. Notice that the exponents on x start at 7 and go down by one each time, and the exponents on y go up by one each time.

10. We can expand binomials involving constants the same way. The coefficients come from Pascal's triangle. The powers of x decrease by one, and the powers on -2 increase by one. We can then multiply the numbers to simplify to the final answer.

11. To recap: The expansion of $(x + y)^n$ has terms whose exponents add to n. It begins with x^n, the powers on x are reduced by one in each step and the powers on y increase by 1 until reaching y^n. The coefficients can be calculated from the factorial formula, or read directly from Pascal's triangle.